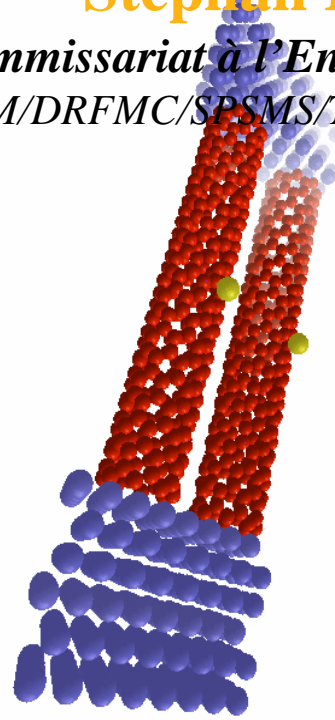


*Quantum Transport in Post-CMOS Molecular
Material and Devices:
Carbon Nanotubes & Semiconducting Nanowires*

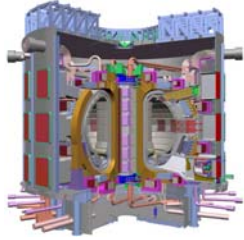


Stephan Roche

*Commissariat à l'Énergie Atomique
DSM/DRFMC/SPSMS/Theoretical Group*



Commissariat à l'Énergie Atomique in Y2006



ITER: *Mastering nuclear fusion*

The main goal of ITER is the study of burning plasmas, i.e. plasmas where heating is mainly provided by the alpha particles created from fusion reactions.



TERA10: *Supercomputing leading facility*

Europe's largest supercomputer

Rank 5th Top500 (august 2006)

Performance *60 Teraflops of computing power, 27 Terabytes of memory
1 Petabytes of disc space with a throughput of 100GB/s*



NEUROSPIN: *From Physics to the Human Brain*

pushing as far as possible the current limits of Magnetic Resonance Imaging (MRI) and spectroscopy to study the central nervous system, from mice to humans.



MINATEC:

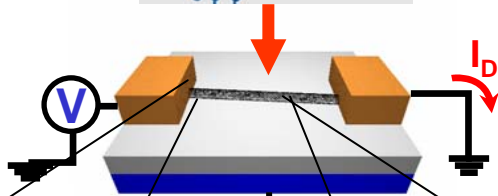
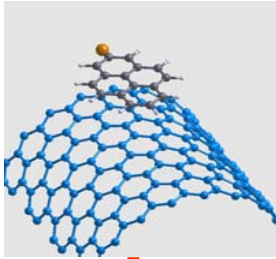
European leading Centre for innovation in micro & nanotechnology



TNT2006 !

POST-CMOS molecular materials & devices

Carbon Nanotubes -- Semiconducting Nanowires -- Graphene



Nanodevices performances

nanoelectronics

optoelectronics

Novel Physical behaviors

spintronique,...

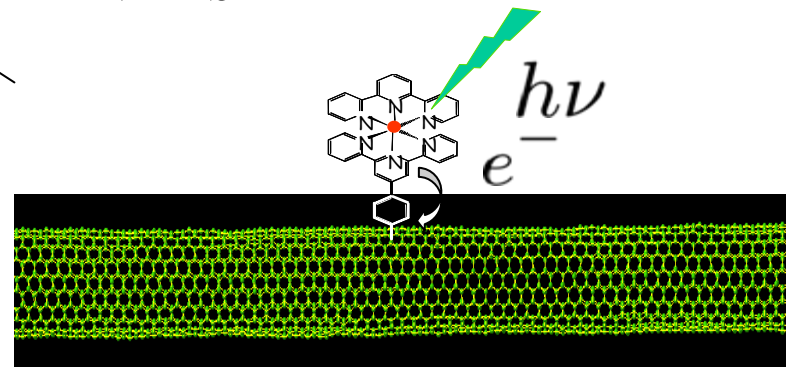
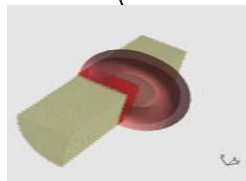
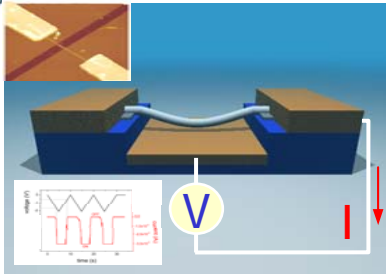
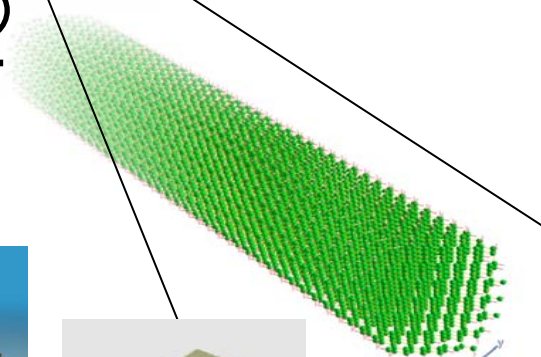
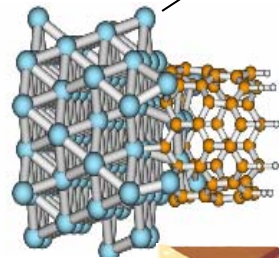
Potential for novel functionalities

(bio)-chemical Sensing

molecular memories,

photo-switches,...

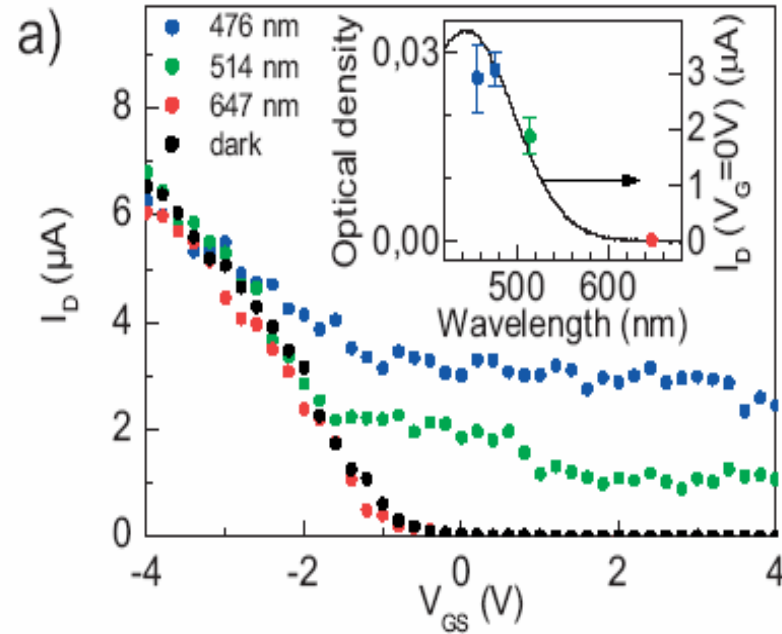
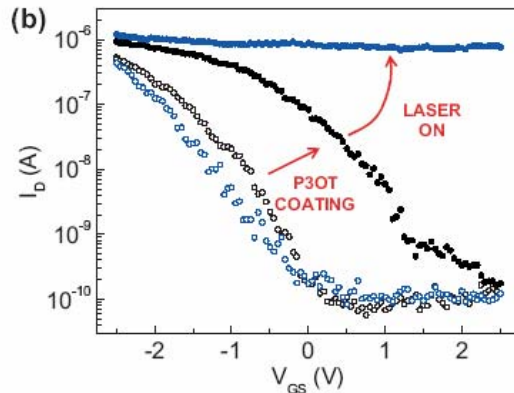
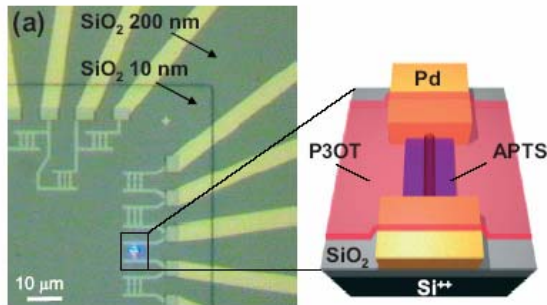
NEMS



DOI: 10.1002/adma.200601138

Optoelectronic Switch and Memory Devices Based on Polymer-Functionalized Carbon Nanotube Transistors**

By Julien Borghetti, Vincent Derycke,* Stéphane Lenfant, Pascale Chenevier, Arianna Filoramo, Marcelo Goffman, Dominique Vuillaume, and Jean-Philippe Bourgoin



Quantum Transport Modelling & Simulation

Our strategy : *Development of quantum transport approaches to tackle fundamental physics in realistic models (predictability efficiency)*

This implies : *Order N methods*

Ab-initio & sophisticated tight-binding models

Pushing the limits beyond mean field and DFT...

This TALK

Coherent Transport

Intrinsic transport / linear response regime (Kubo formula)

- *Elastic mean free path in disordered systems (charge mobilities, etc..)*
- *Weak localization regimes (magnetoresistance patterns)*
- *Conductance scaling*

Quantum Transport for open systems (Landauer-Büttiker formula)

- *Ballistic transport and contact dependent transmission phenomena*

Out-of-equilibrium regimes & Inelastic Transport

Poisson-Schrödinger solver (*Field-effect Transistor physics*)

QM treatment of el-ph coupling (*beyond mean field*),..

Quantum Transport by Kubo approach

Solving time dependent
Schrödinger equation

$$|\Psi(t)\rangle = \exp(-i\frac{Ht}{\hbar})|\Psi(0)\rangle$$

Efficient order N real space methods



$$\frac{e^2 h}{\Omega} \text{Tr}[\hat{V}_x \delta(E-H) \hat{V}_x \delta(E-H)] = \lim_{t \rightarrow \infty} \text{Tr}[\delta(E-H) \frac{(\hat{X}(t) - \hat{X}(0))^2}{t}]$$

Quantum dynamics
(conduction regimes)

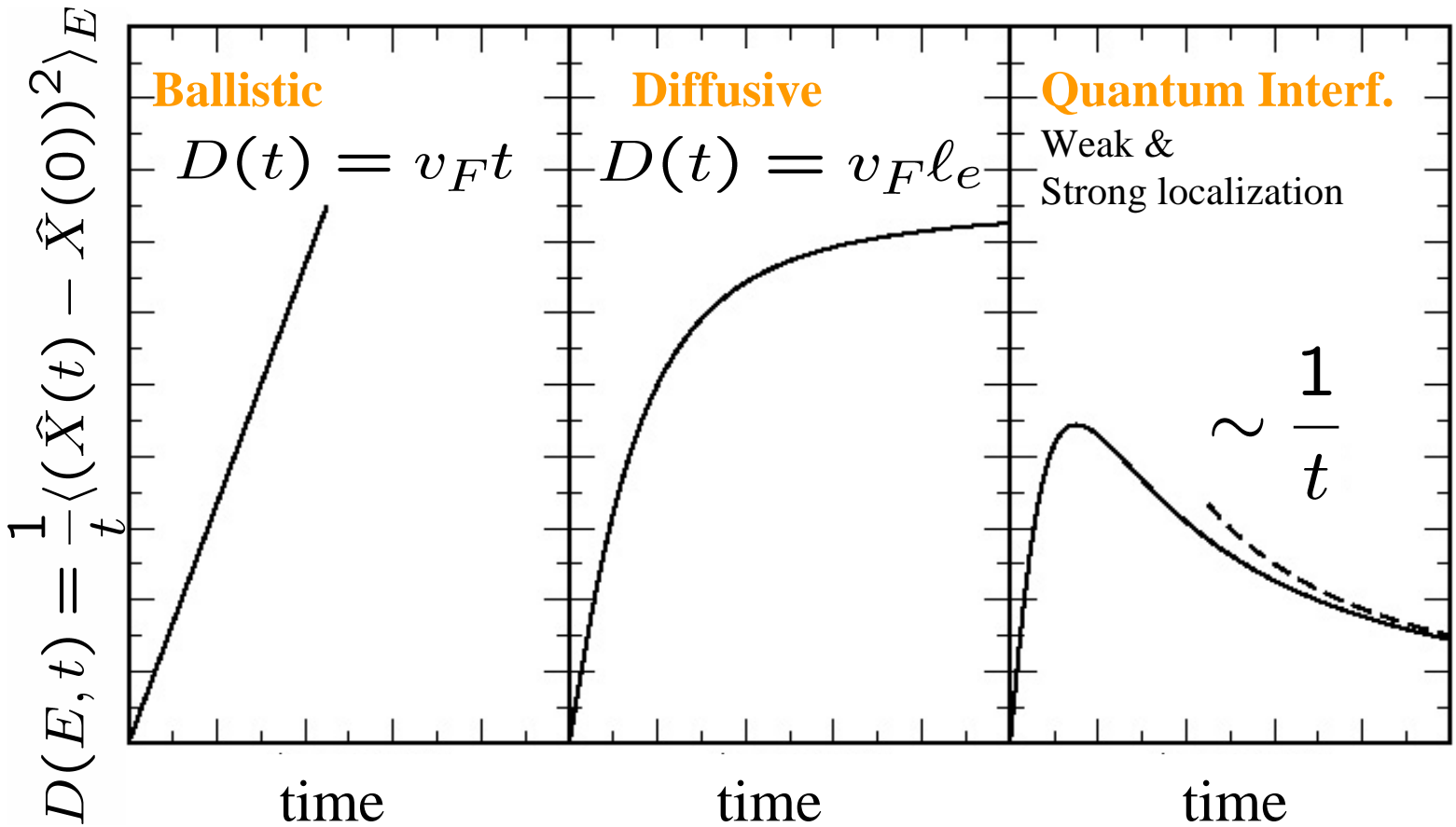
Kubo Conductance
(conductance scaling)

$$D(E, t) = \frac{1}{t} \langle (\hat{X}(t) - \hat{X}(0))^2 \rangle_E \quad G(E) = \frac{2e^2}{L_{\text{sys}t}} \lim_{t \rightarrow \tau_L} \text{Tr}[\delta(E-H) D(E, t)]$$

S.R, D. Mayou, **Phys. Rev. Lett. 79, 2518 (1997)**

S.R, R. Saito, **Phys. Rev. Lett. 87, 246803 (2001)**

Quantum Transport Regimes & Conductance scaling



$$G(E) = \frac{2e^2}{h} N(E)$$

$$G(E) = \frac{2e^2}{h} \frac{l_e}{L_{\text{sys}t}}$$

$$G(E) = \frac{2e^2}{h} \left(\frac{l_e}{L_{\text{sys}t}} - \delta\sigma \right)$$

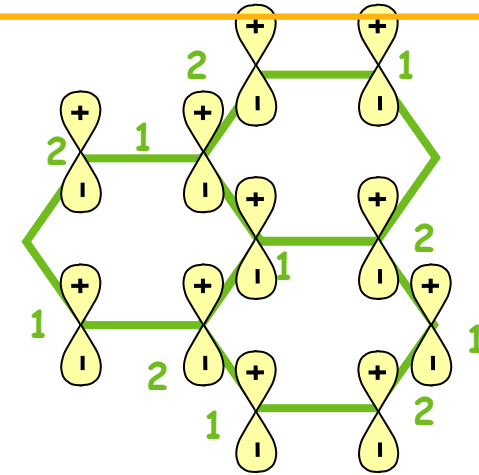
$$G(E) = \frac{2e^2}{h} \exp\left(-\frac{l_e}{L_{\text{sys}t}}\right)$$

Effective π electrons-model

Hybrid molecular orbitals

Cohesion $s, p_x, p_y \equiv \sigma$

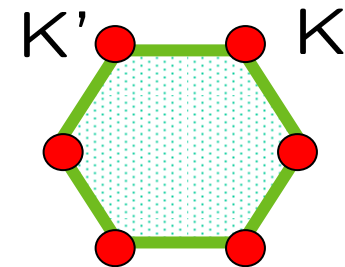
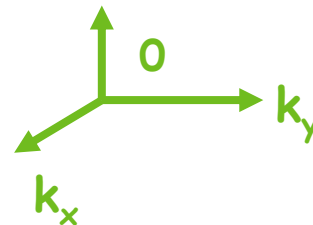
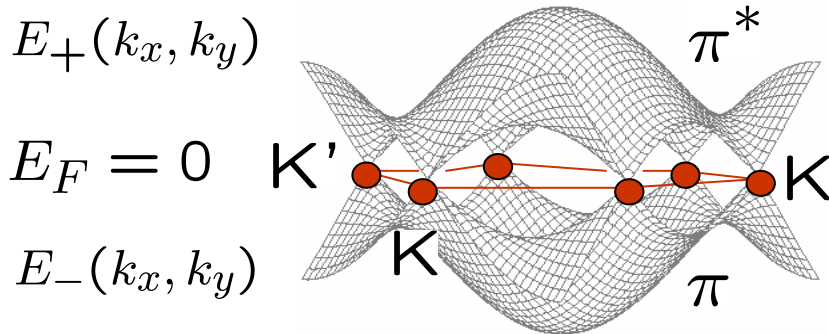
Electronic properties close to E_F $p_z \equiv \pi$



2 atoms/ unit cell γ_0 coupling between orbitals p_z

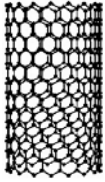
$$H(\vec{k}) = \begin{bmatrix} 0 & f(\vec{k}) \\ f^*(\vec{k}) & 0 \end{bmatrix} \quad f(\vec{k}) = \gamma_0 \sum_{\alpha} e^{i\vec{k} \cdot \vec{\tau}_{\alpha}}$$

$$E_{\pm}(k_x, k_y) = \pm \gamma_0 \left(3 + 4 \cos\left(\frac{\sqrt{3}k_x a}{2}\right) \cos\left(\frac{k_y a}{2}\right) + 2 \cos(k_y a) \right)^{1/2}$$



1th Brillouin Zone

Nanotubes: Electronic Properties



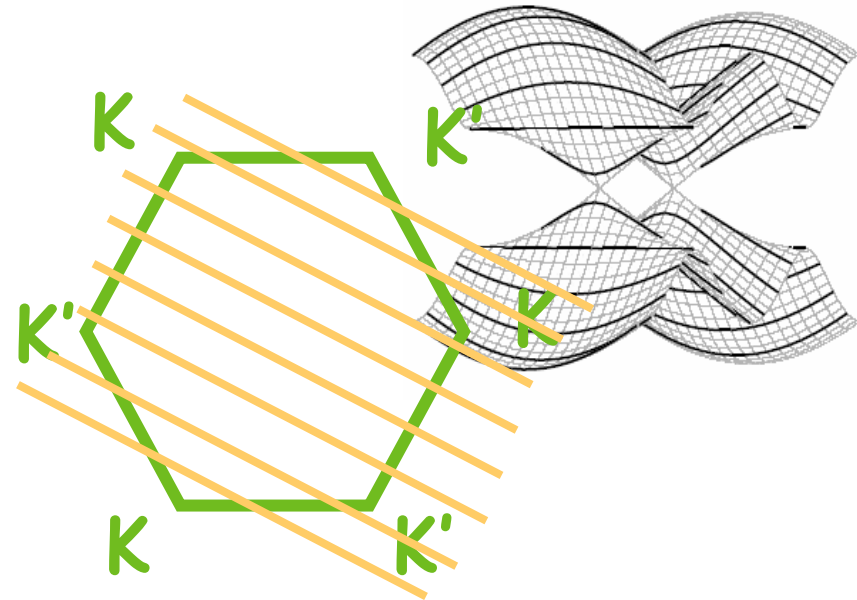
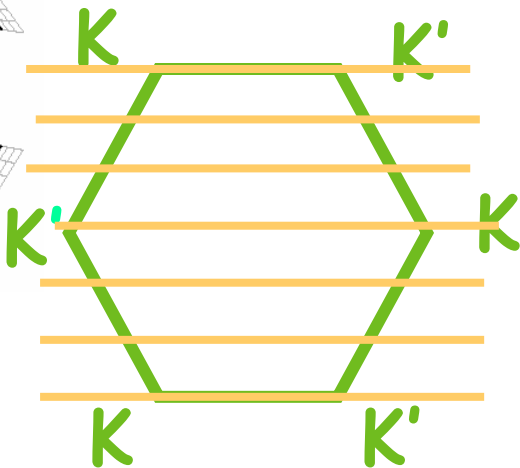
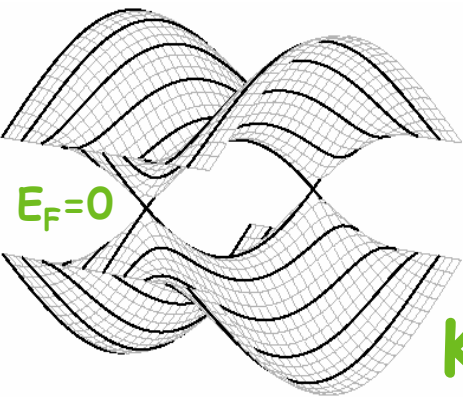
Periodic boundary conditions

$$-\frac{\pi}{|\vec{T}_{(n,m)}|} \leq k_y (= k) \leq +\frac{\pi}{|\vec{T}_{(n,m)}|} \quad k_x = \frac{2\pi q}{|\vec{C}_{(n,m)}|} (q = 1, N)$$

Helicity

$$\vec{C}_{n,n} = n(\vec{a}_1 + \vec{a}_2)$$

$$\vec{C}_{n,m} = (3p \pm 1)\vec{a}_1$$



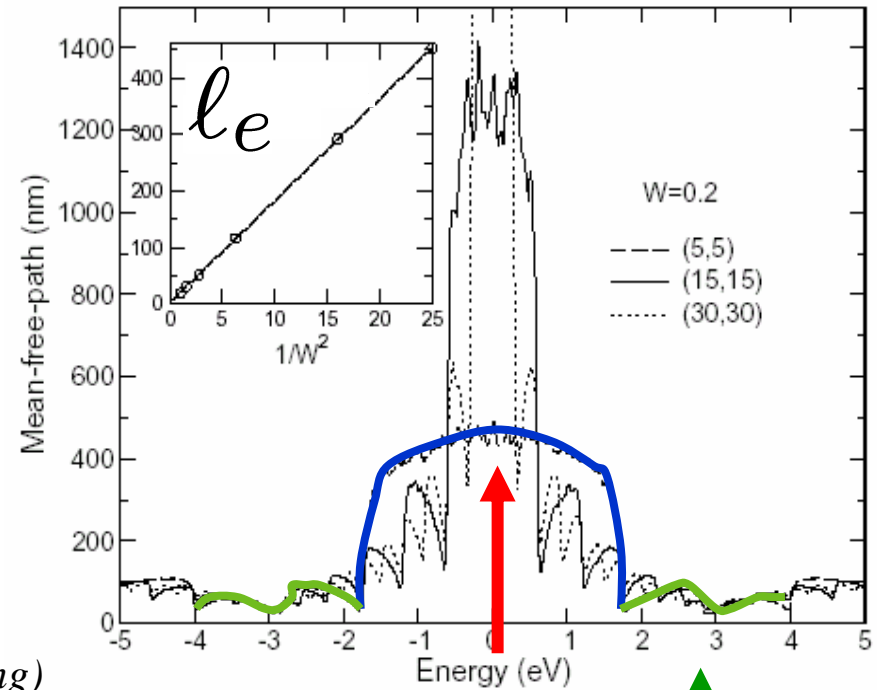
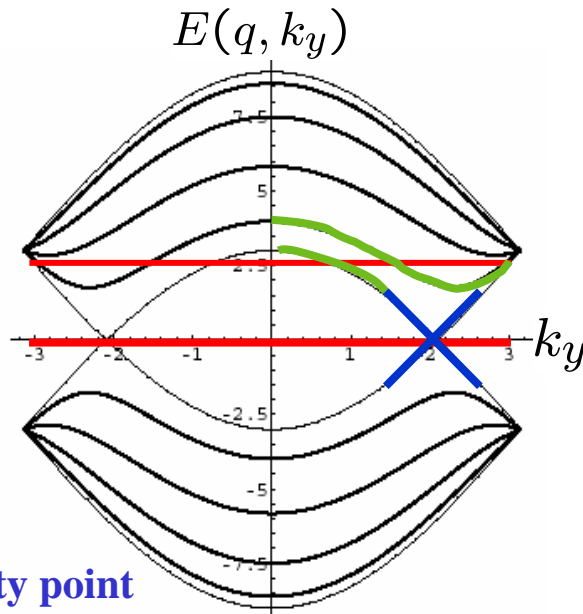
Energy-dependent mean free path



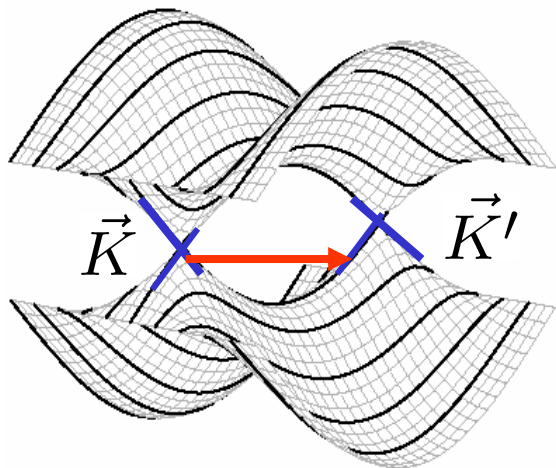
(5,5)
Nanotube

Close to the
charge neutrality point

Fermi Golden Rule (disorder-induced backscattering)



Under Fermi level shift



$$\frac{1}{\tau(E_F)} = \frac{2\pi}{\hbar} |\langle \Psi_{n_1}(k_F) | \hat{U} | \Psi_{n_2}(-k_F) \rangle|^2 \rho(E_F)$$

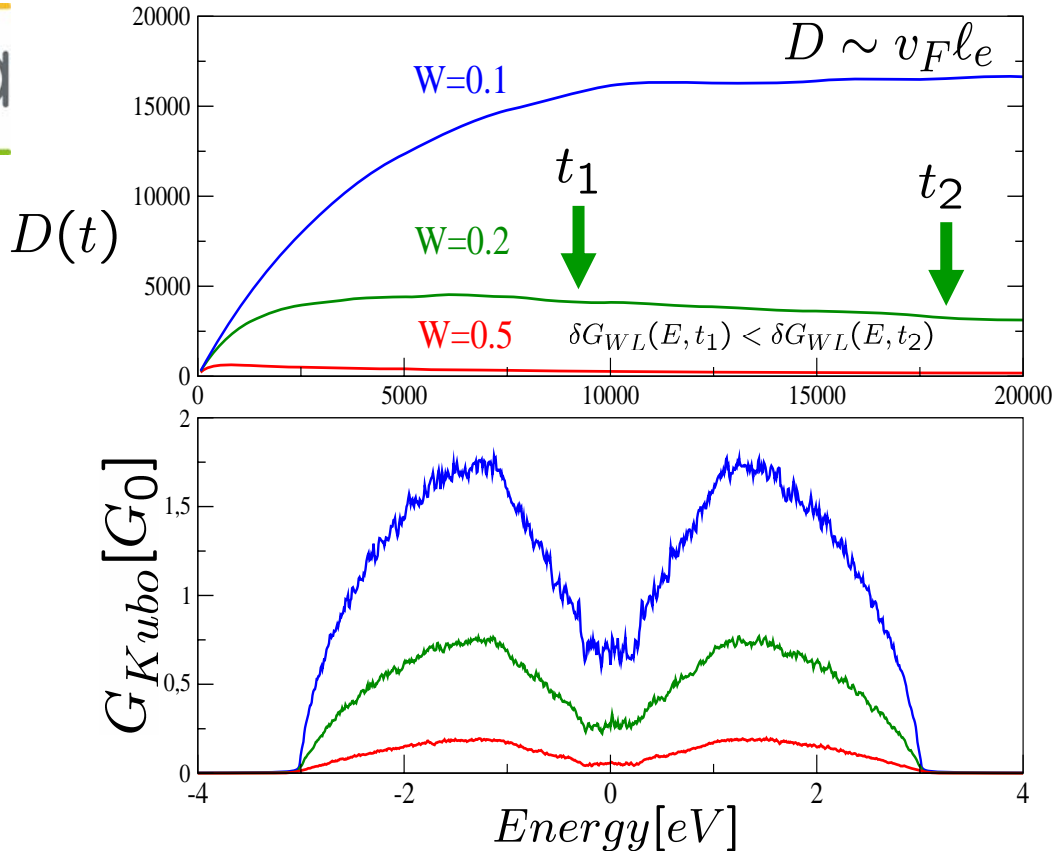
$$l_e = v_F \tau(v_F) \longrightarrow l_e = \frac{18}{\sqrt{3}} \left(\frac{\gamma_0}{W}\right)^2 \|\vec{c}_h\|$$

Anderson-type elastic disorder (W)

F. Triozon, S.R, A. Rubio, D. Mayou, **Phys. Rev. B 69, 121410 (2004)**

Weak localization & coherence length scaling

Nanotube: (10,10) Anderson type static disorder strength = W



→ Saturation regime
Mean free path

$$\ell_e$$

$$\tau_{class}(E) = \frac{2e^2}{h} N_{\perp} \frac{\ell_e(E)}{L(E, t)}$$

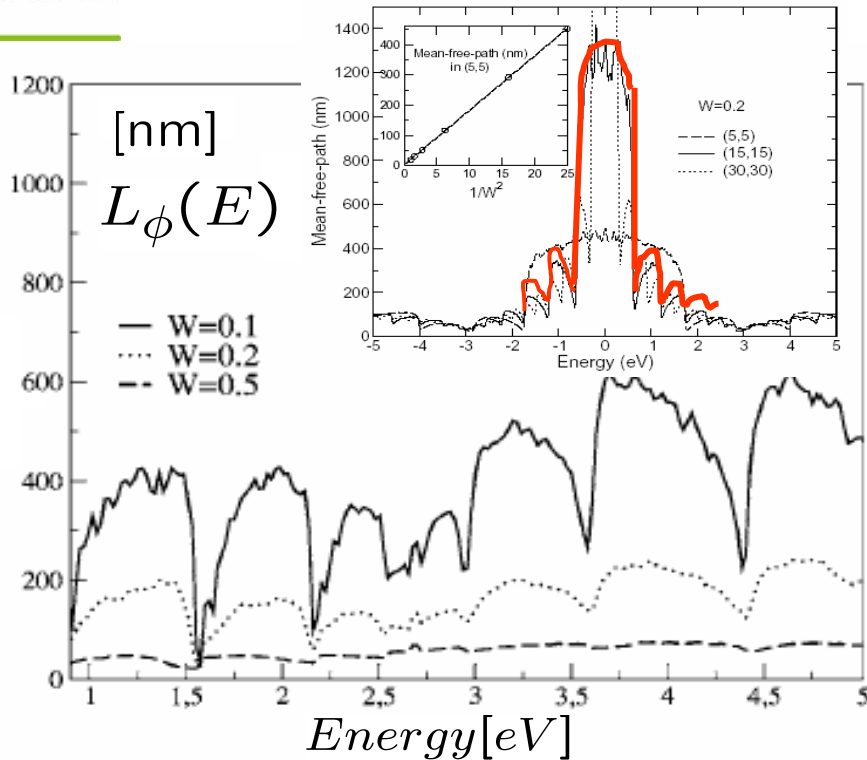
Scaling properties
Coherence length

$$\delta G_{WL}(E) = \frac{2e^2}{h} N_{\perp} \frac{\ell_e(E)}{L(E, t)} - G_{Kubo}(E)$$

→ $L_{\phi}(E)$

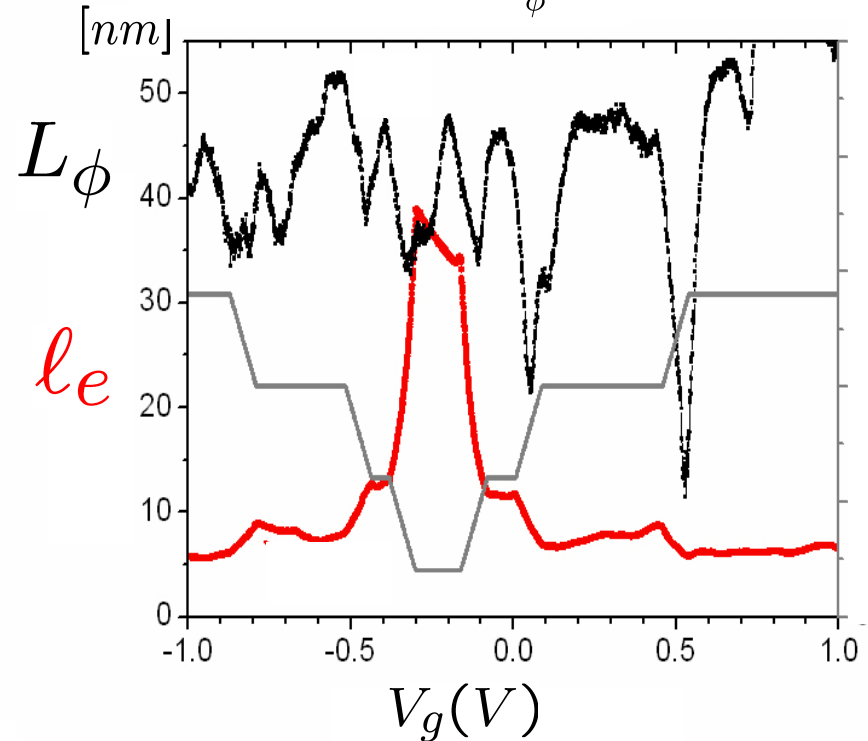
Coherence lengths/time in the diffusive regime

$$\delta G_{WL}(E) = \frac{2e^2}{h} \frac{L_\phi(E)}{L(E, t)}$$



Experimental results
Exploring Weak localization
under gate voltage

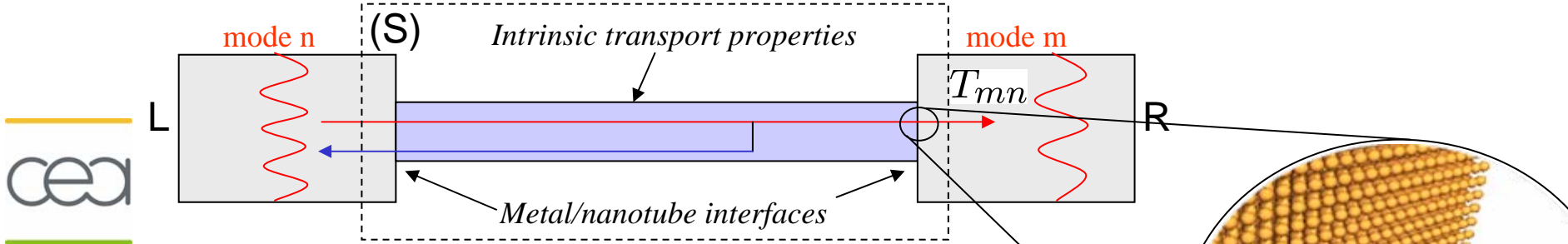
$$\delta G_{WL} = -A \frac{e^2}{\pi \hbar L} \left(\frac{1}{L_\phi^2} + \frac{W^2 e^2 B^2}{3 \hbar^2} \right)^{-1/2}$$



S.R, J. Jiang, F. Triozon, R. Saito,
Phys. Rev. B 72, 113410 (2005)

B. Stojetz et al., **PRL 94, 186802 (2005)**

Quantum Transport by the Landauer-Büttiker Approach



$$\frac{dI}{dV} = \frac{2e^2}{h} \sum_{m,n} T_{mn} = \mathcal{G}(E) = \frac{2e^2}{h} \text{Tr}[\Gamma_L G^r \Gamma_R G^a]$$

$$G^{r,a}(E) = \frac{1}{E - \mathcal{H}_{Sys} - (\Sigma_L^{r,a}(E) + \Sigma_R^{r,a}(E))}$$

Self-energy due to metal/nanotube coupling

Finite size system → \mathcal{H}_{Sys}

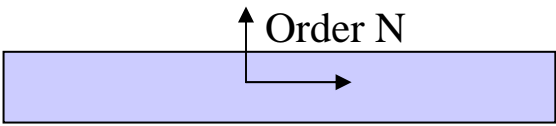
$$\Sigma_{L,R}^{r,a}(E) = V_{L,R-Sys}^\dagger g_{L,R}^{r,a}(E) V_{L,R-Sys}$$

$$\Gamma_{L,R}(E) = i[\Sigma_{L,R}^r(E) - \Sigma_{L,R}^a(E)]$$

- *) *Decimation recursive techniques*
- *) *Order N (bi-orthogonalization process) and continuous fraction expansion*

$$\mathcal{H} = \mathcal{H}_{Sys} + \Sigma_L^r + \Sigma_R^r \quad \text{Effective non-symmetric H}$$

$$\mathcal{G} = \frac{2e^2}{h} \sum_{\alpha,\beta,\alpha',\beta'} \langle \beta | \Gamma_R | \alpha \rangle \langle \alpha | G^r | \alpha' \rangle \langle \alpha' | \Gamma_L | \beta' \rangle \langle \beta' | G^a | \beta \rangle$$



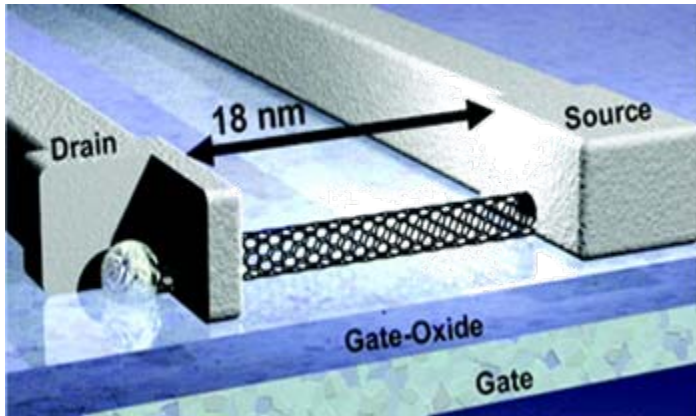
Ballistic regimes & field effect transistors

Semiconducting CNTs

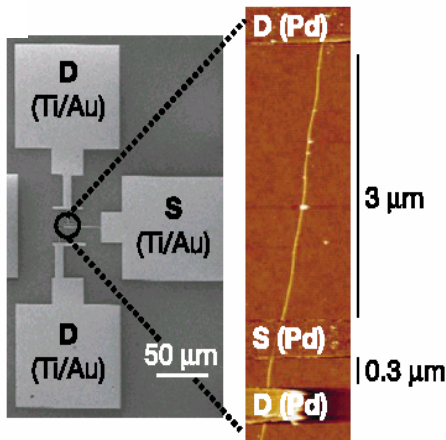
Ballistic transport $\ell_e \geq L_{\text{canal}}$

$$G(E) = \frac{2e^2}{h} N_{\perp}(E)$$

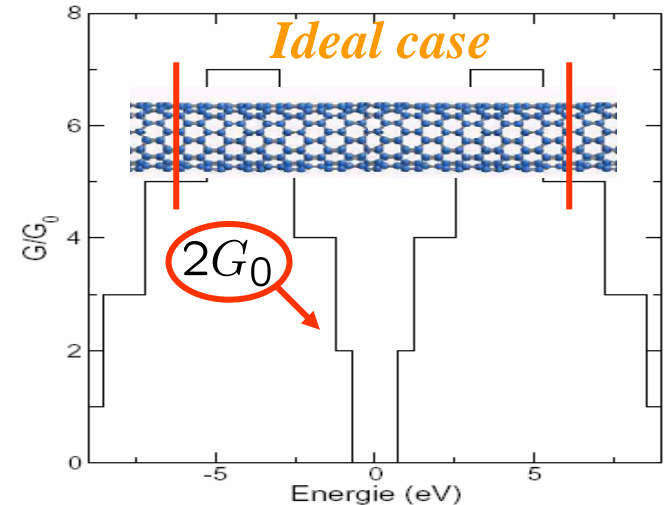
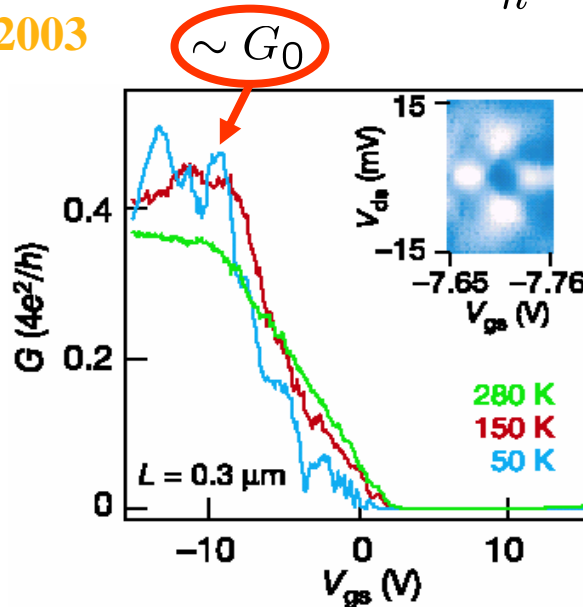
$$\frac{2e^2}{h} = G_0 = (13\text{k}\Omega)^{-1}$$



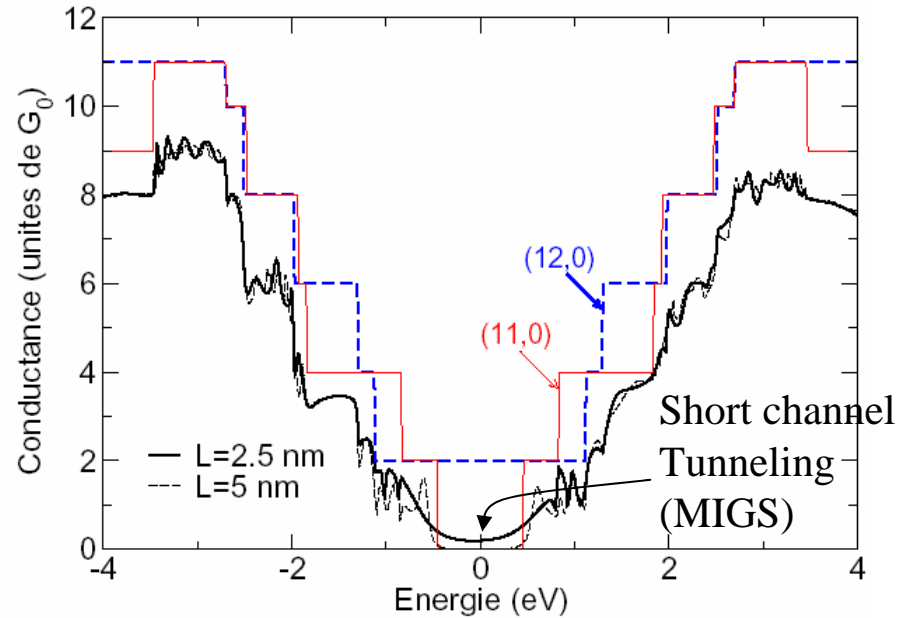
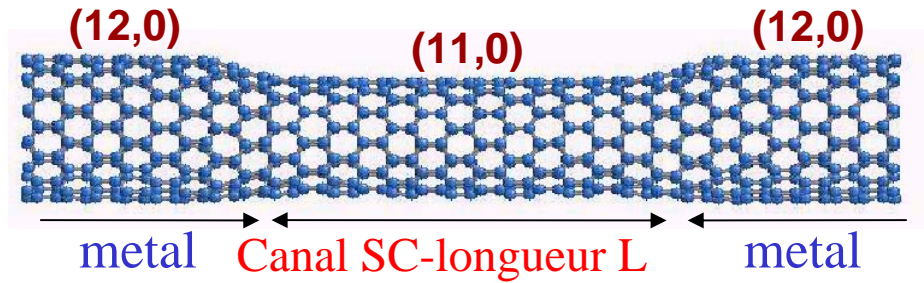
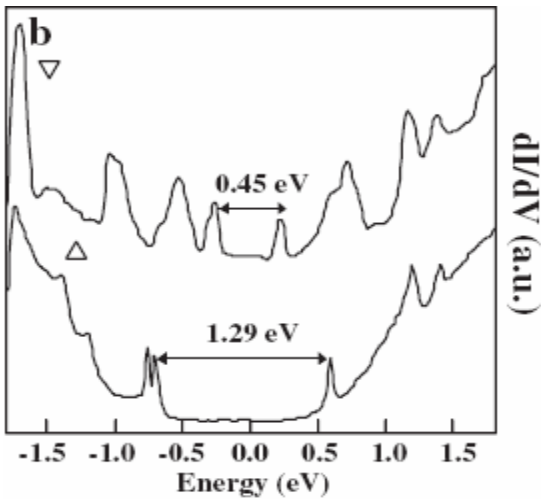
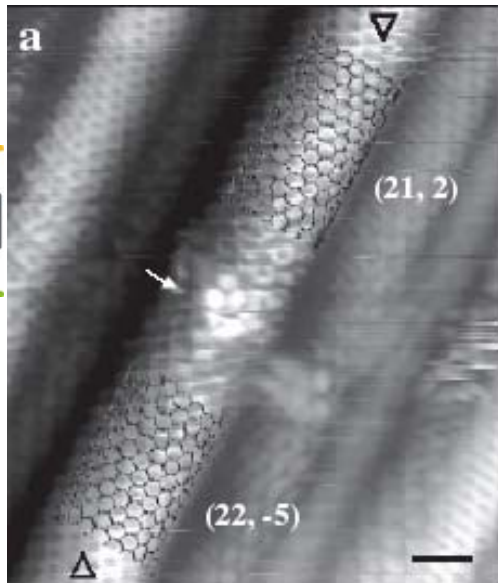
A. Javey et al., *Nature* 2003



Ohmic contact
(Palladium, p-type)



Metal/SC/Metal intramolecular junctions



$T < 1$ quantum reflections at interfaces

T. Odom, J. Huang, C.M. Lieber,
J. Phys. C. 14 R145 (2002)

F.Trioizon, Ph. Lambin, SR, **Nanotechnology (2005)**

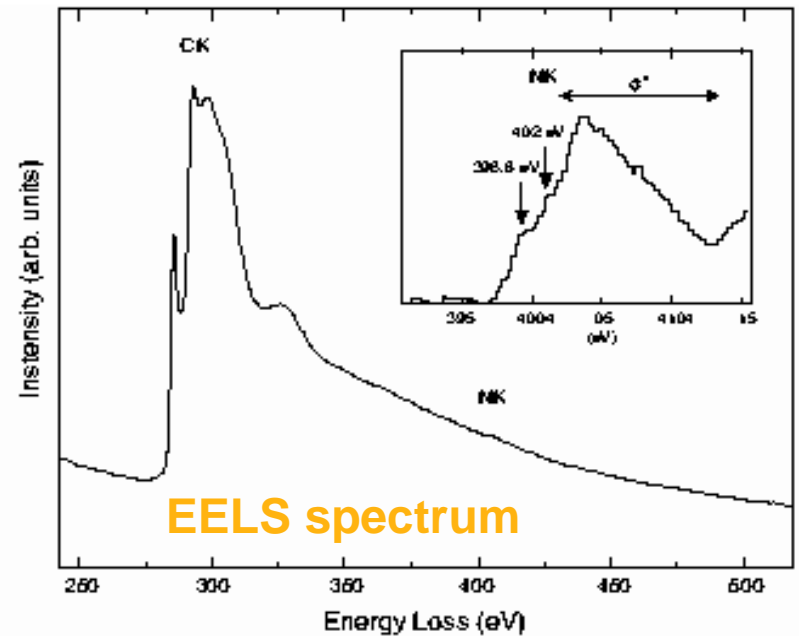
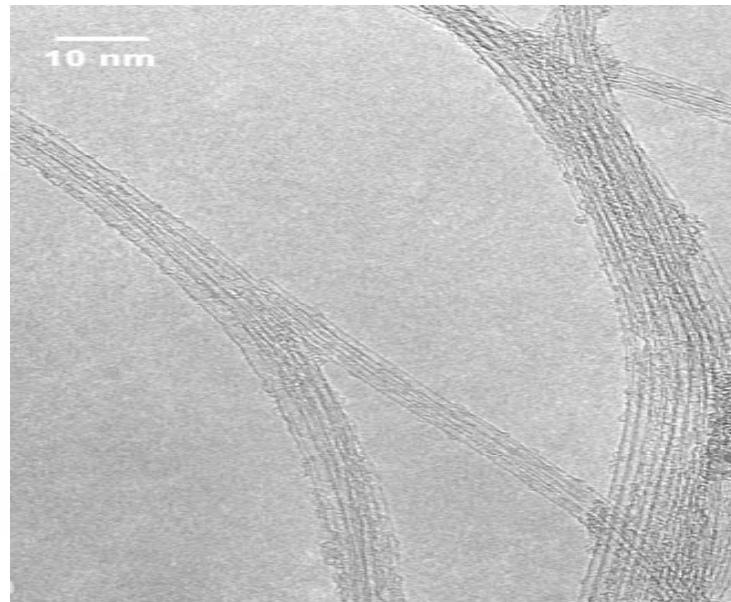
Properties of chemically modified Carbon nanotubes

- Substitutions Nitrogen (n) or Boron (p)-*
- Physisorption potassium-*
- Physisorption benzene, azulene,..*

Doped nanotubes with Nitrogen or Boron

Incorporation of N_2 (gas) during the synthesis

Bundles of Nitrogen-doped Single-wall nanotubes



M. Glerup *et al.*

Chem. Phys. Lett. 387, 193 (2004)

Fig. 3. EEL core electron K-shell spectra of CN_x nanotube bundles (sample 4). The nanotubes are doped with around 1 at.% nitrogen. For the C-K edge well defined π^* and σ^* fine structure features are observed which are evidences of sp^2 -hybridisation in graphitic structures. The inset is a magnification of the N-K edge.

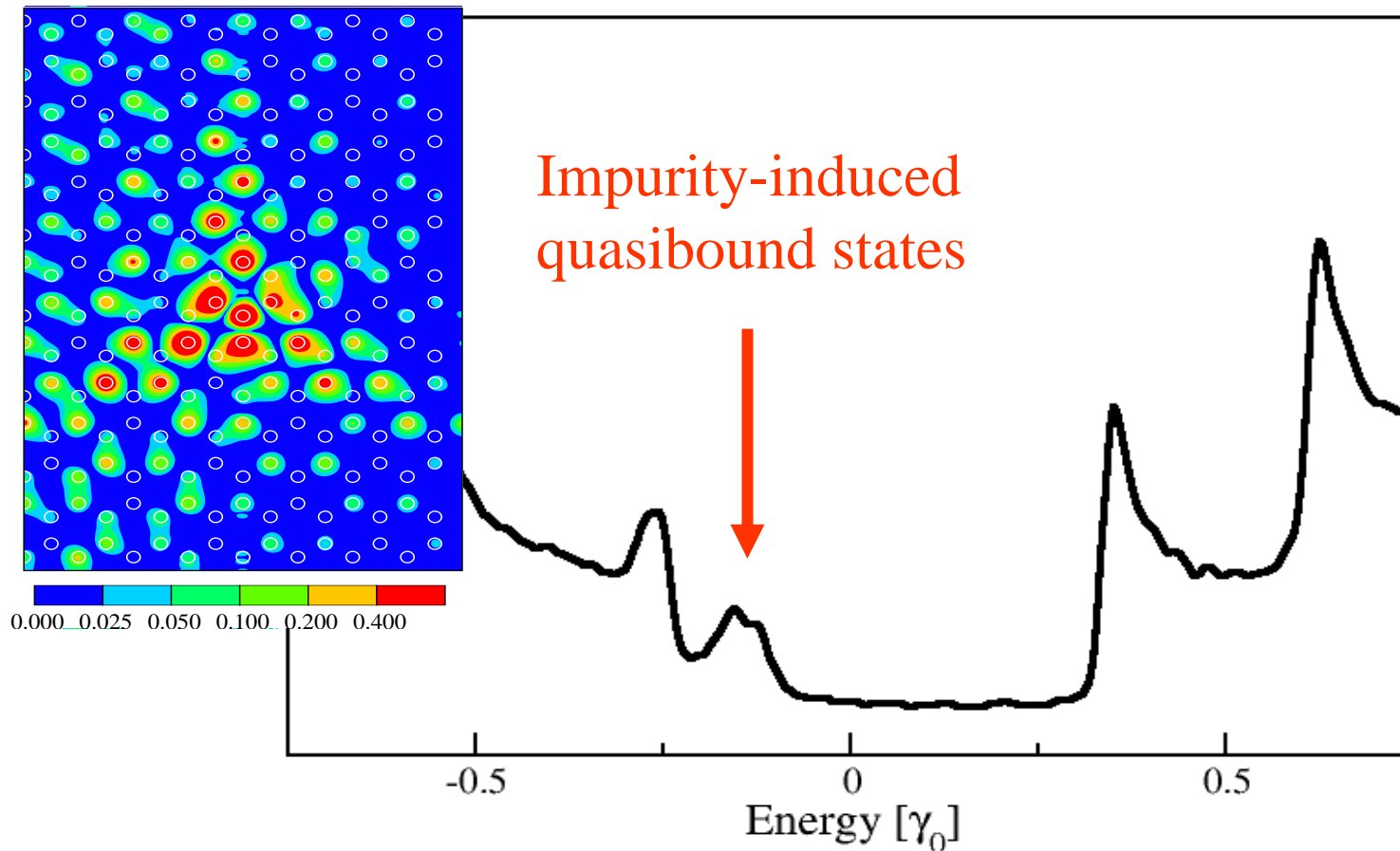
Substitution de bore dans un nanotube



modèle *Zone Folding* (ZF)

Nanotube (10,10)

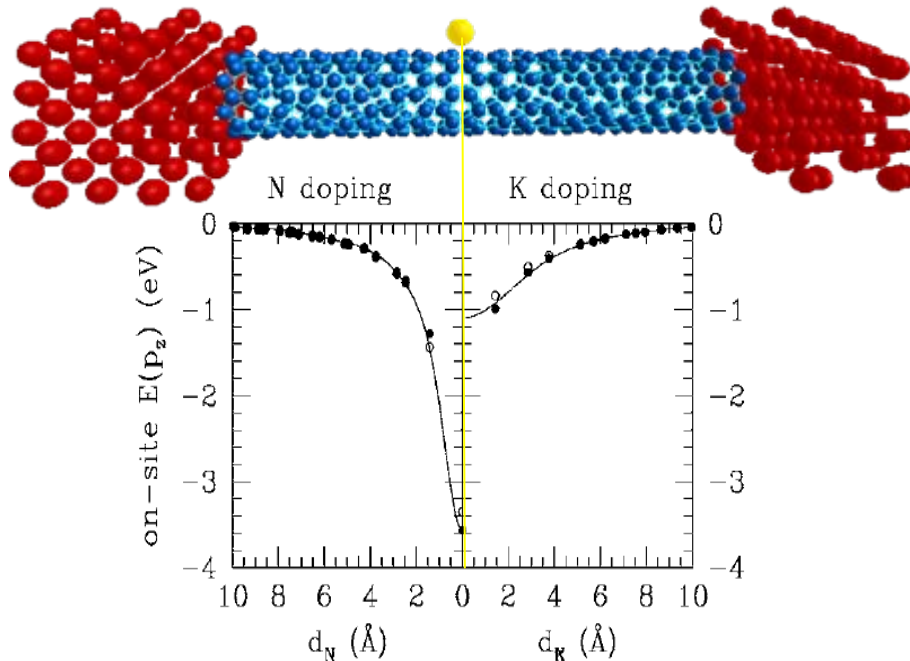
BORON: 0.5 %



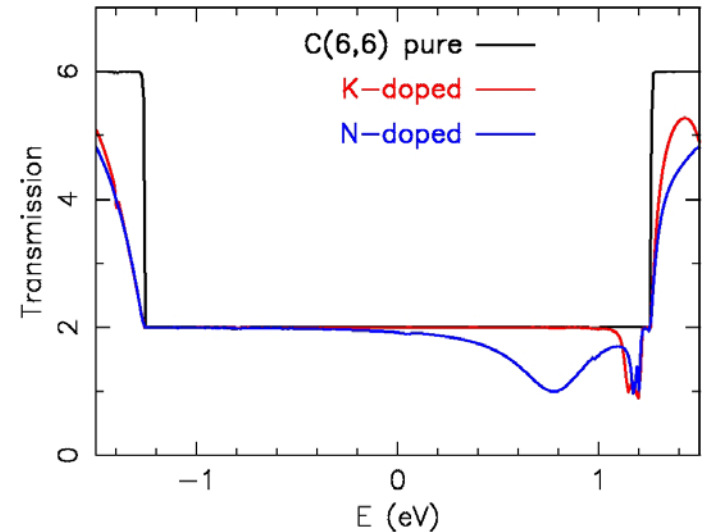
Chemically Doped Carbon Nanotubes

Combining

- ab-initio* calculations (accuracy of energetics at atomistic level)
- effective tight-binding (performing large scale studies)



Single-impurity case (X. Blase)
Transport calculation (SIESTA)



Gaussian potential
Effective model

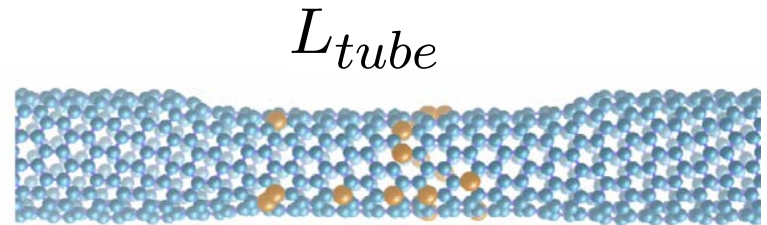
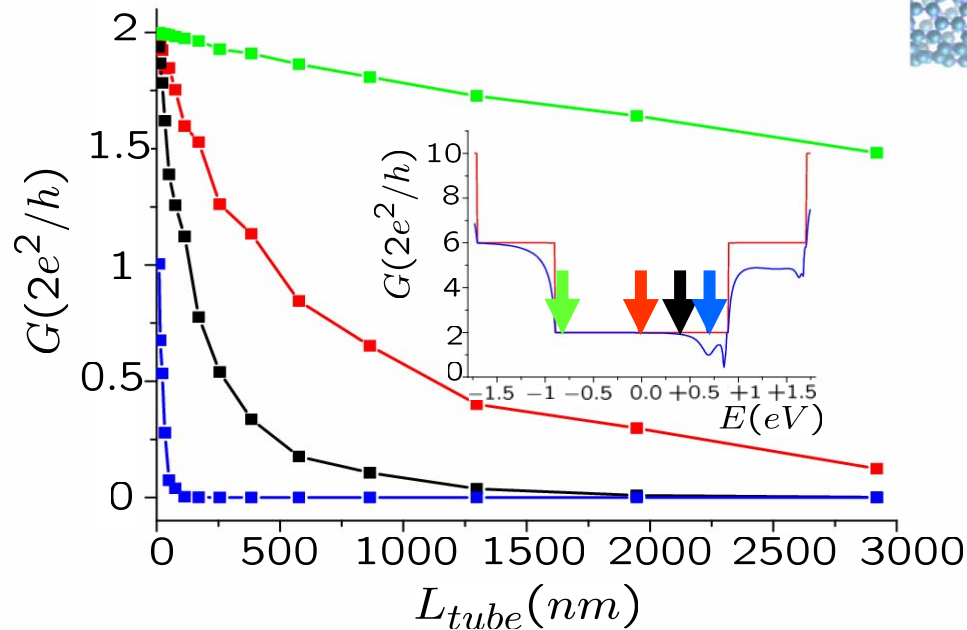
$$\tilde{\mathcal{H}}_{\text{NT}}(\alpha) = -\tilde{\gamma}_0 \sum_{\langle i,j \rangle} \hat{c}_i^\dagger \hat{c}_j + hc + \sum_i \tilde{V}_{\alpha i} \hat{c}_i^\dagger \hat{c}_i$$

Scaling study of quantum conductance

Chemically disordered nanotubes with length up to several microns

$$G(\epsilon) = G_0 T(\epsilon) \quad T(\epsilon) = \text{Tr}\{\hat{t}_{LR}(\epsilon)\hat{t}_{LR}^\dagger(\epsilon)\}$$

Metallic nanotube (10,10)
Doping N = 0.1%



Quasi-ballistic

Diffusive

elastic mean free path + QIE

Strong localization

Statistical Analysis of T(E) & Transport Regimes

Quasi-ballistic

$$L \ll l_e \ll l_\phi$$

$$\langle T \rangle = N_\perp(\epsilon)$$

Diffusive

$$l_e \ll L \ll l_\phi$$

$$\langle T \rangle = N_\perp(\epsilon) \frac{l_e(\epsilon)}{L}$$

Localized

$$\xi \ll L \ll l_\phi$$

$$\langle \ln T \rangle = -\frac{L}{\xi}$$

$$\langle T \rangle = \frac{N_\perp(\epsilon)}{1 + \frac{L}{l_e(\epsilon)}}$$

↓

elastic mean free path

l_e

Thouless relationship (agreement with RMT)

$E(\text{eV})$	$l_e(\text{nm})$	$l_e^K(\text{nm})$	$\xi(\text{nm})$	ξ/l_e	l_e^K/l_e
-0.78	8424 ± 95	≈ 6766	/	/	≈ 0.8
0.00	466 ± 16	≈ 460	841 ± 38	1.8 ± 0.1	≈ 1.0
0.35	126 ± 9	≈ 91	170 ± 4	1.3 ± 0.1	≈ 0.7
0.69	8.2 ± 0.8	≈ 6.4	8.75 ± 0.02	1.1 ± 0.1	≈ 0.8
1.25	24.7 ± 2.7	≈ 17	97.2 ± 1.7	3.9 ± 0.4	≈ 0.7
$E(\text{eV})$	$l_e(\nu = 1.21)$	/	$\xi(\nu = 1.21)$	$\frac{\xi}{l_e}(\nu = 1.21)$	/
0.69	192.1 ± 7.5	/	396.8 ± 12.4	2.1 ± 0.1	/
1.25	19.9 ± 2.4	/	194.6 ± 5.4	9.8 ± 1.2	/

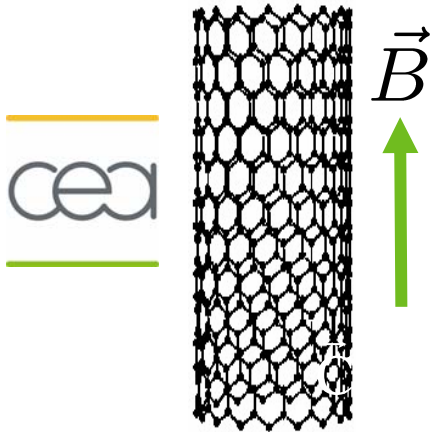
$$\beta = 1$$

$$\xi = \frac{1}{2} \{ \beta(N_\perp - 1) + 2 \} l_e$$

$$\beta = 2$$

breaking TRS (perp B-field)

Aharonov-Bohm Effects on Bandstructure



$\{\vec{C}_h/|\vec{C}_h|, \vec{T}/|\vec{T}|\}$

Landau gauge

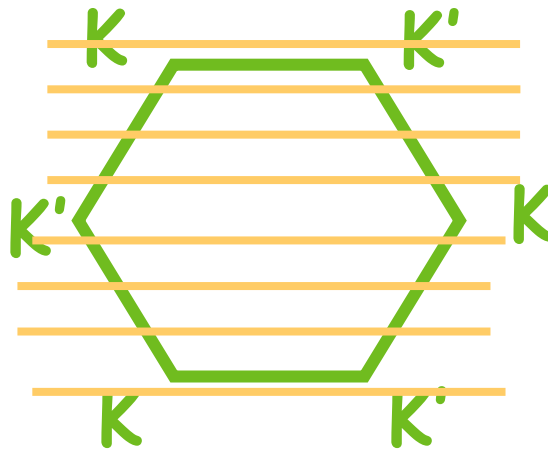
$$\vec{A} = (\phi/|\vec{C}_h|, 0)$$

Wavefunction

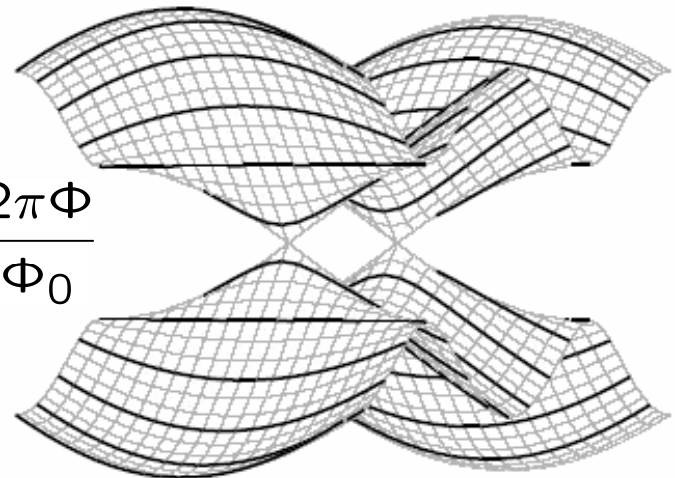
$$\Psi \sim e^{ik_y y} e^{i(k_x x + \frac{e}{\hbar} \int \vec{A} \cdot d\vec{r})}$$

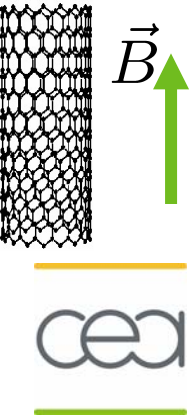
$$\begin{aligned} \Delta\varphi_{\vec{r}, \vec{r}'} &= \int_0^1 (\vec{r}' - \vec{r}) \cdot (\vec{A}(\vec{r} + \lambda[\vec{r}' - \vec{r}])) d\lambda \\ &= i(x - x')\phi/|\vec{C}_h| \end{aligned}$$

$$\delta\vec{k}(\phi) \cdot \vec{\kappa}_x = \delta\vec{k}(0) \cdot \vec{\kappa}_x + 2\pi\phi/(\phi_0|\vec{C}_h|)$$



$$k_x = \frac{2\pi q}{|\vec{C}_h|} + \frac{2\pi\Phi}{\Phi_0}$$



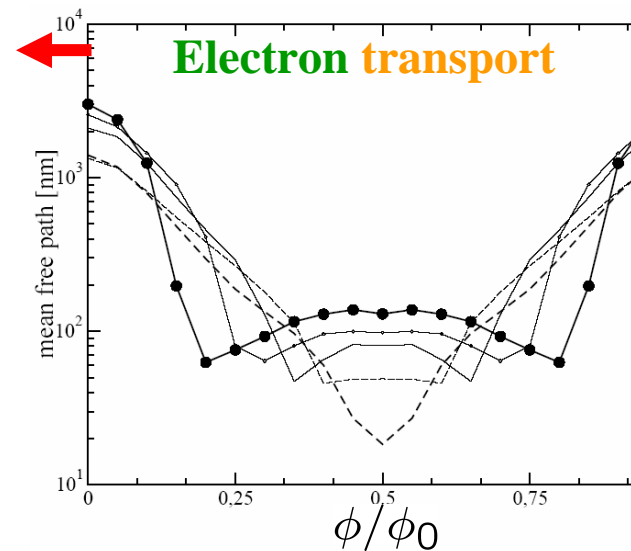
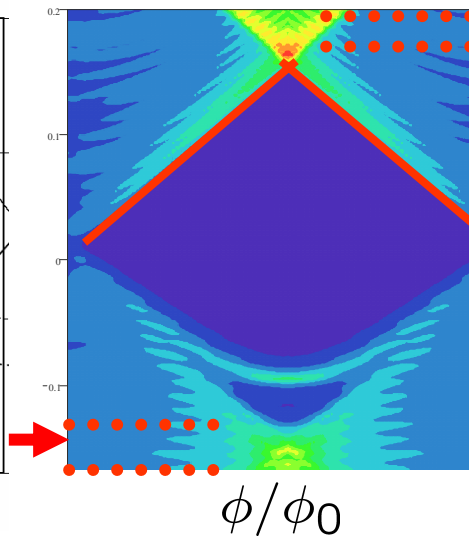
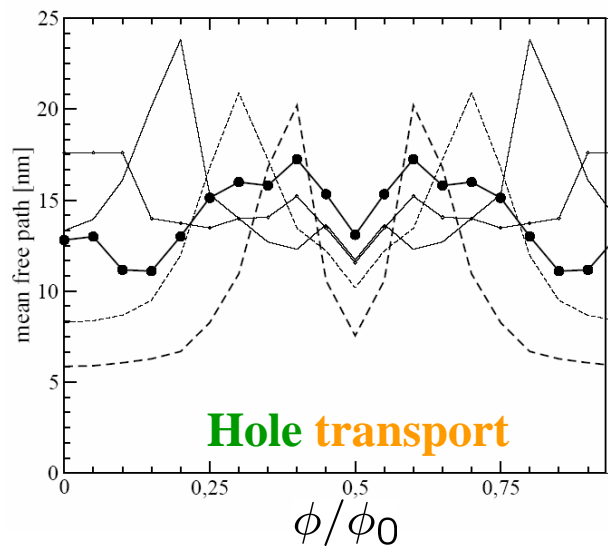


Transport under B and Aharonov-Bohm effects

Chemical impurities

-Breaking of electron-hole symmetry

$$D(t) = v(E)l_e \sim v^2(E)\tau$$

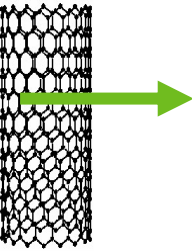


Holes: $l_e \in [5\text{nm}, 25\text{nm}]$
Electrons: $l_e \in [200\text{nm}, 4\mu\text{m}]$



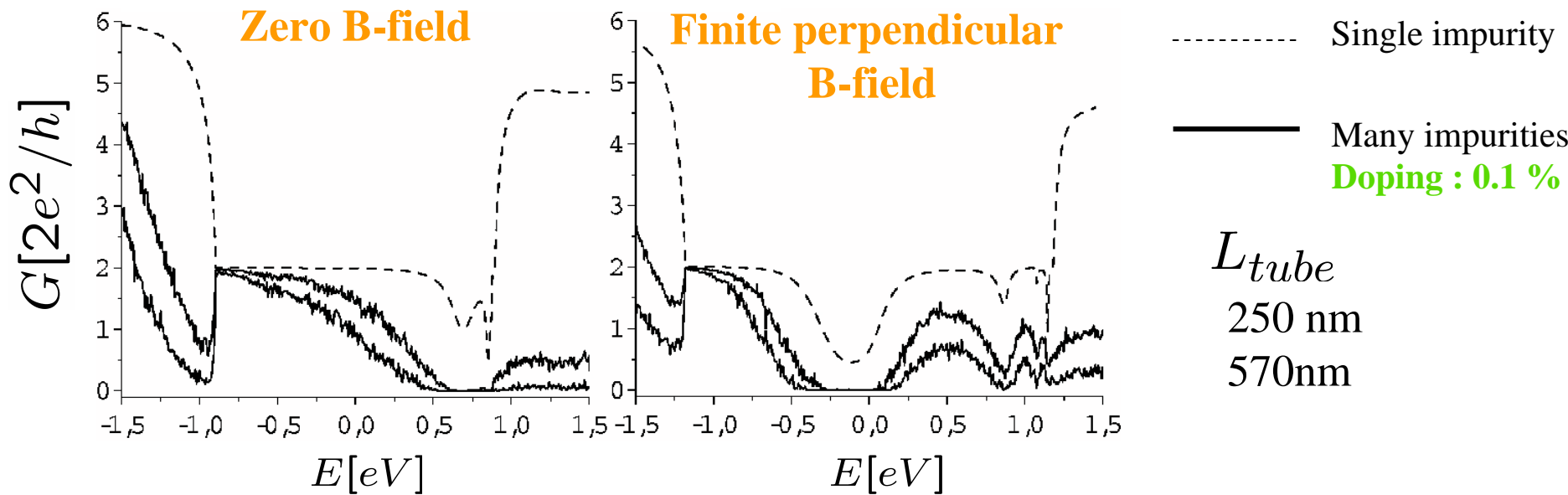
P-doped but hole current *is vanishingly small* compared to electron current

Colossal Magnetoresistance fluctuations (Nitrogen doped)



$$\tilde{\mathcal{H}}_{\text{NT}}(\Omega, \vec{A}) = -\tilde{\gamma}_0 \sum_{\langle i,j \rangle} e^{-i\theta_{ij}(\vec{A})} \hat{c}_i^\dagger \hat{c}_j + \text{hc} + \sum_{\alpha \in \Omega} \sum_i \tilde{V}_{\alpha i} \hat{c}_i^\dagger \hat{c}_i$$

$$\theta_{ij}(\vec{A}) = \frac{1}{Bl_B^2} \int_i^j \vec{A} \cdot d\vec{l} \quad \ell_B = \sqrt{\frac{\hbar}{eB}}$$



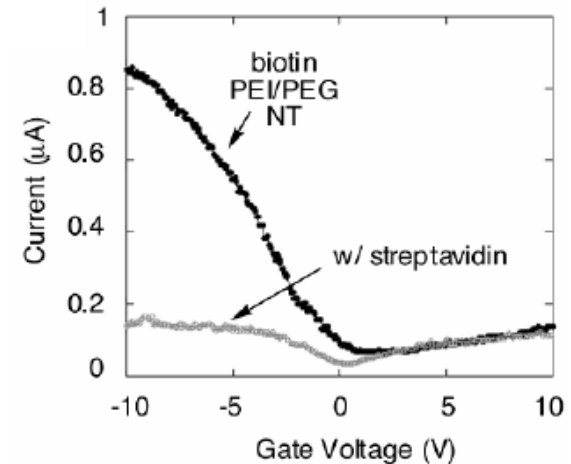
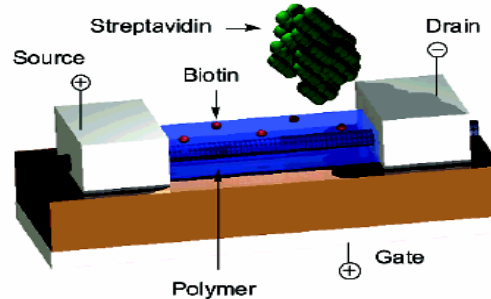
Nanoscale sensing ?

Sensitiveness and selectivity / molecular adsorption



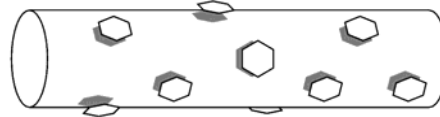
*Interactions Proteines ,
pH, enzymatic activity,...*

**A. Star et al.,
Nanoletters 3, 459 (2003)**



Physisorption

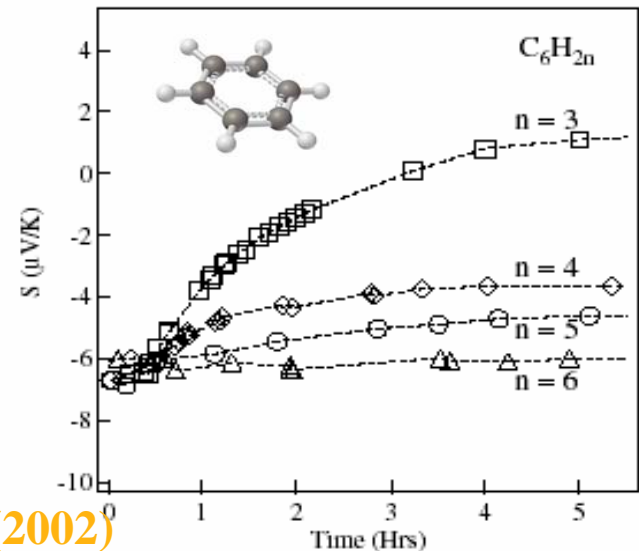
Any measurable effects ?



*Giant changes of electrical thermopower
under benzene physisorption*

$$S = -\frac{\pi^2 k_B}{3eG} \frac{dG(E)}{dE} \Big|_{E_F}$$

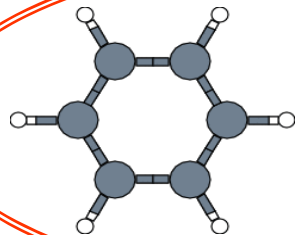
G. Sumanakera et al. Phys. Rev. Lett. 89, 166801 (2002)



π -conjugated

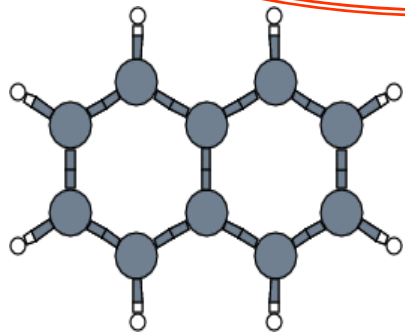


Cycle aromatic



Benzene (C_6H_6)

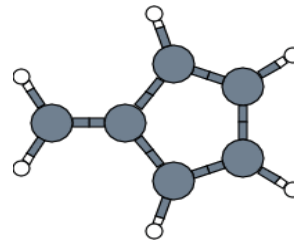
$E_g = 5.15 \text{ eV}$



Naphthalene ($C_{10}H_8$)

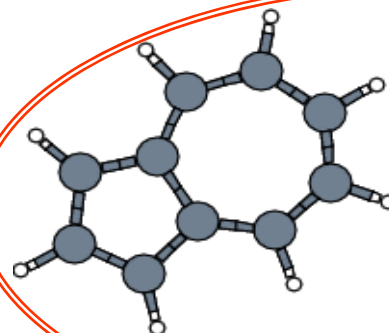
$E_g = 3.41 \text{ eV}$

Non-aromatic



Fulvene (C_6H_6)

$E_g = 2.46 \text{ eV}$



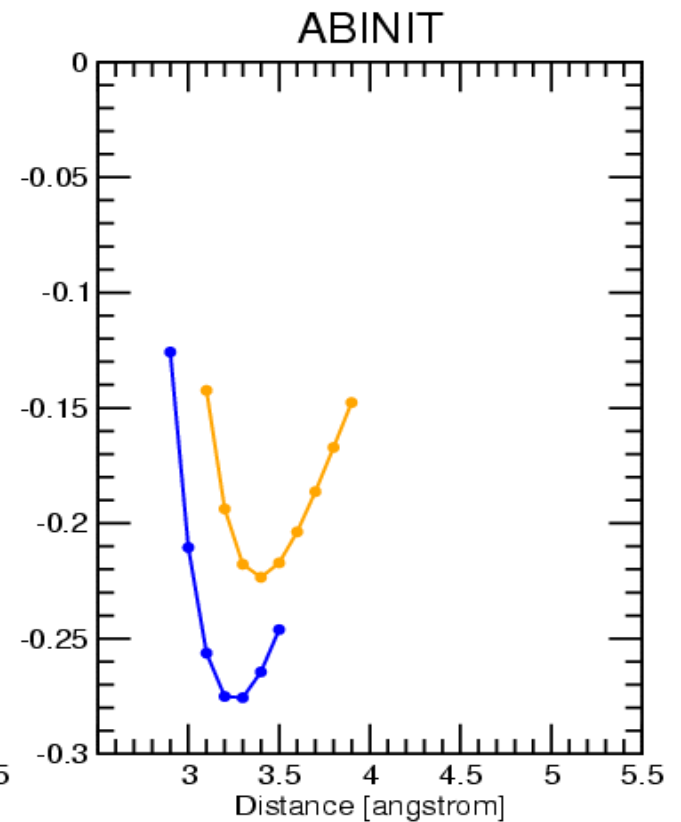
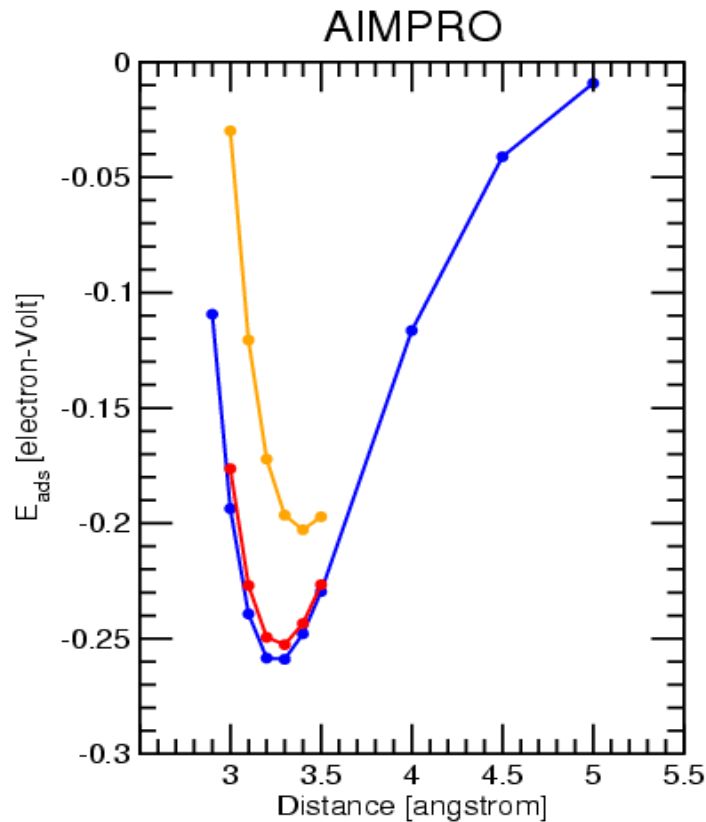
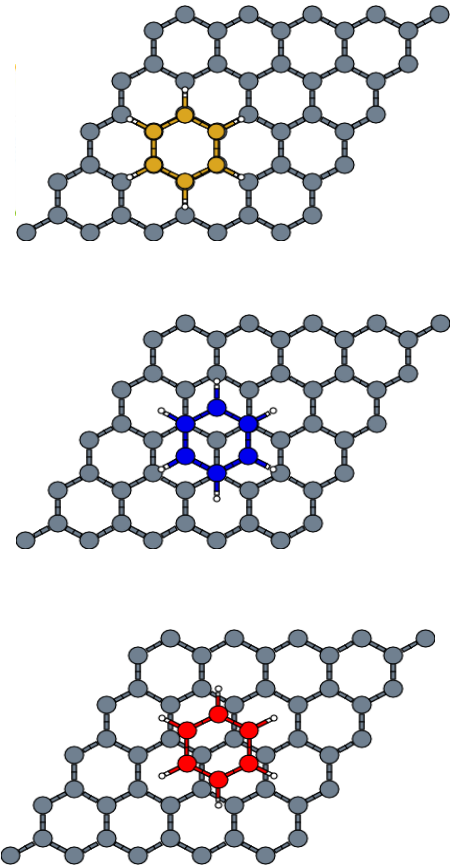
Azulene ($C_{10}H_8$)

$E_g = 2.07 \text{ eV}$

Ab-initio calculation

DFT-LDA (structure + electronic properties)

Equilibrium positions



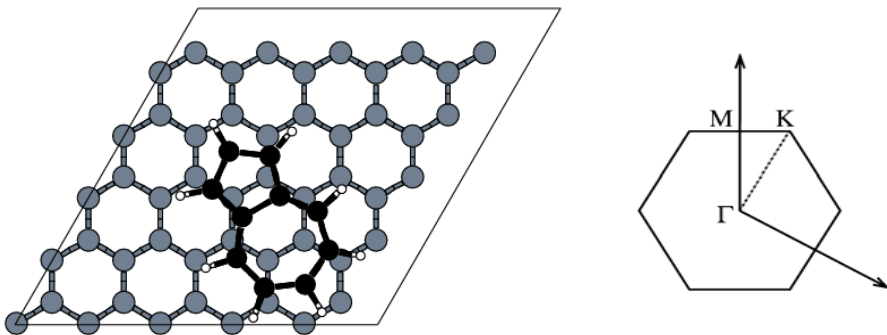
<http://aimpro.ncl.ac.uk>

<http://www.abinit.org>

S. Latil (Univ. Namur)

Electronic properties : multiscale method

ab-initio (DFT)

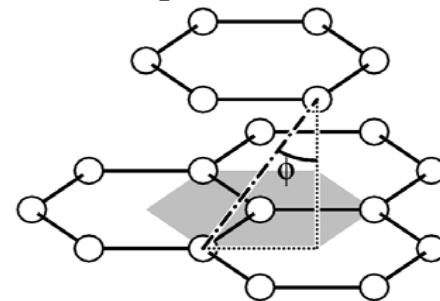
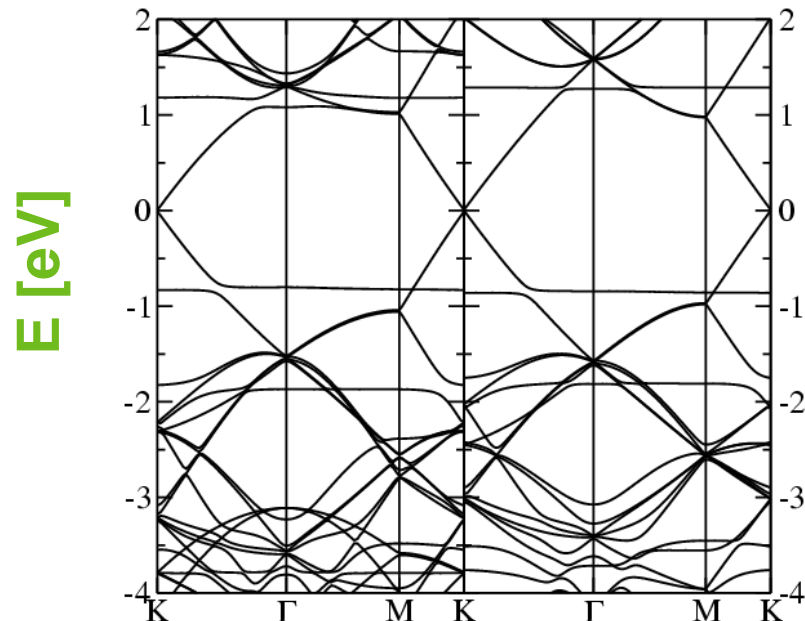


Effective model

$|\pi_i\rangle =$ onsite orbital i , $energy = \varepsilon_i$

$$H = \sum_{\langle i,j \rangle} \varepsilon_i |\pi_i\rangle \langle \pi_i| - \gamma_0 \sum_{\langle i,j \rangle} |\pi_j\rangle \langle \pi_i| - \sum_{\langle i,j \rangle} \beta_{ij} |\pi_j^{mol}\rangle \langle \pi_i^{CNT}| + C.C$$

DFT Tight-binding parametrization

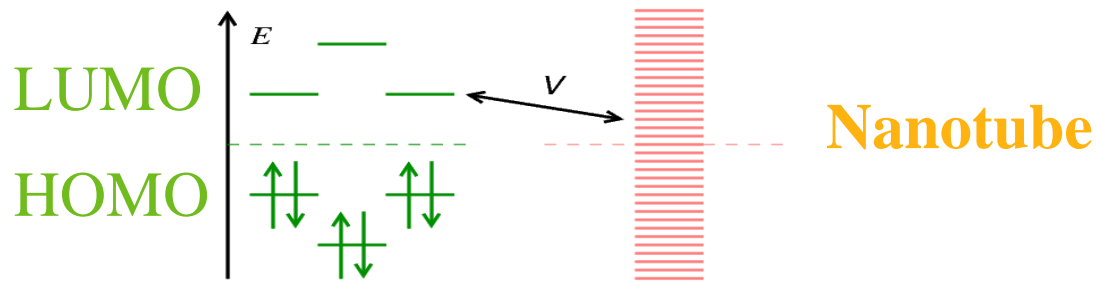


$$\beta_{ij} = \gamma_0 \cos(\phi_{ij}) e^{-(d_{ij}-\delta)/L}$$

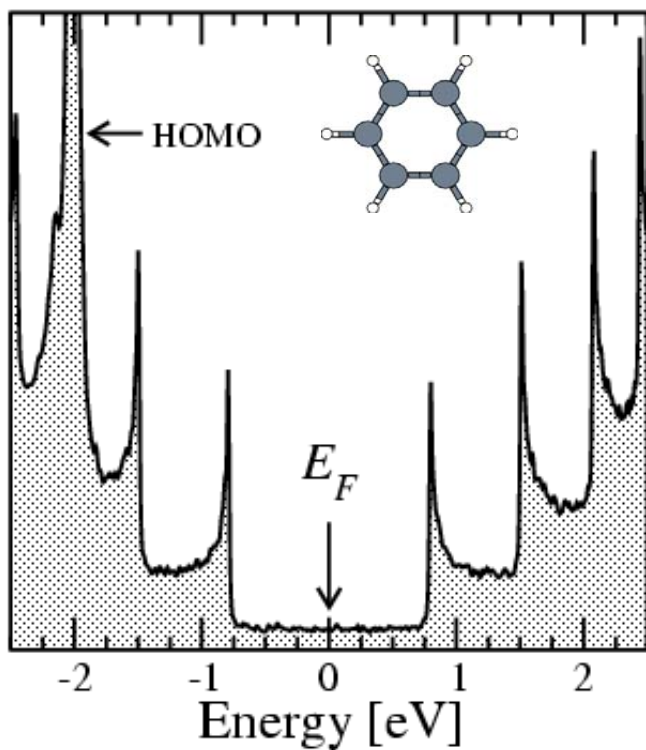
Electronic properties of hybrid systems



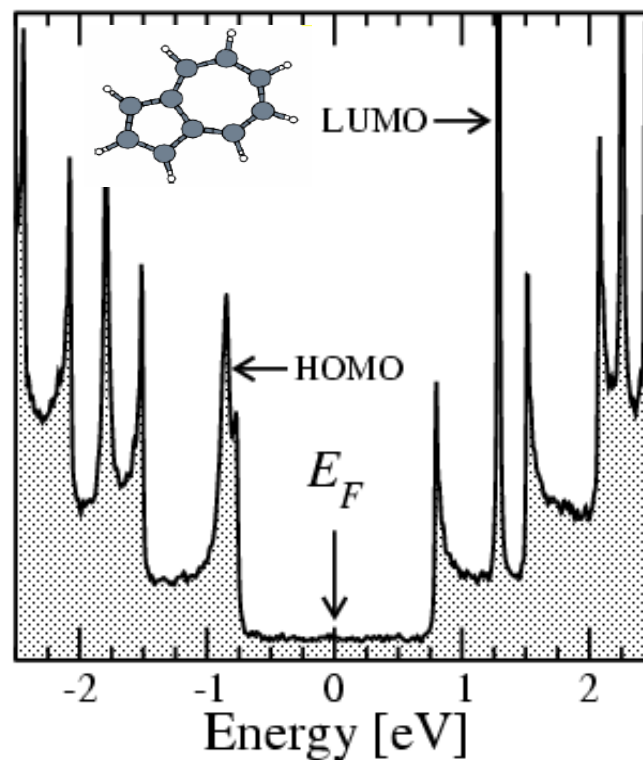
Molecules



Density of states



Density of states

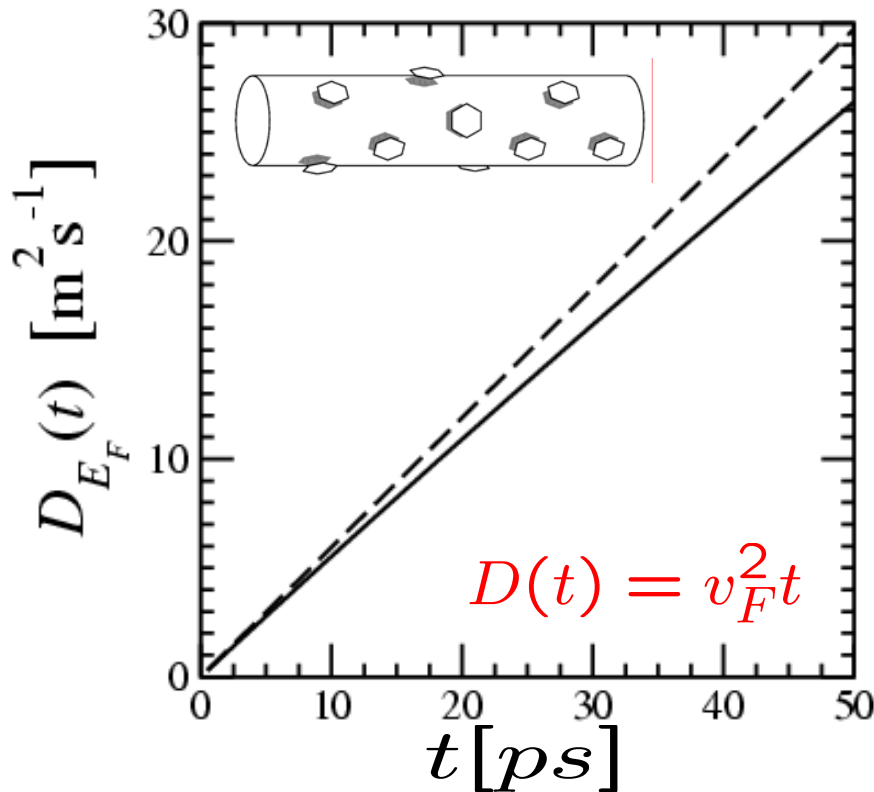


Molecular dependent transport scaling properties

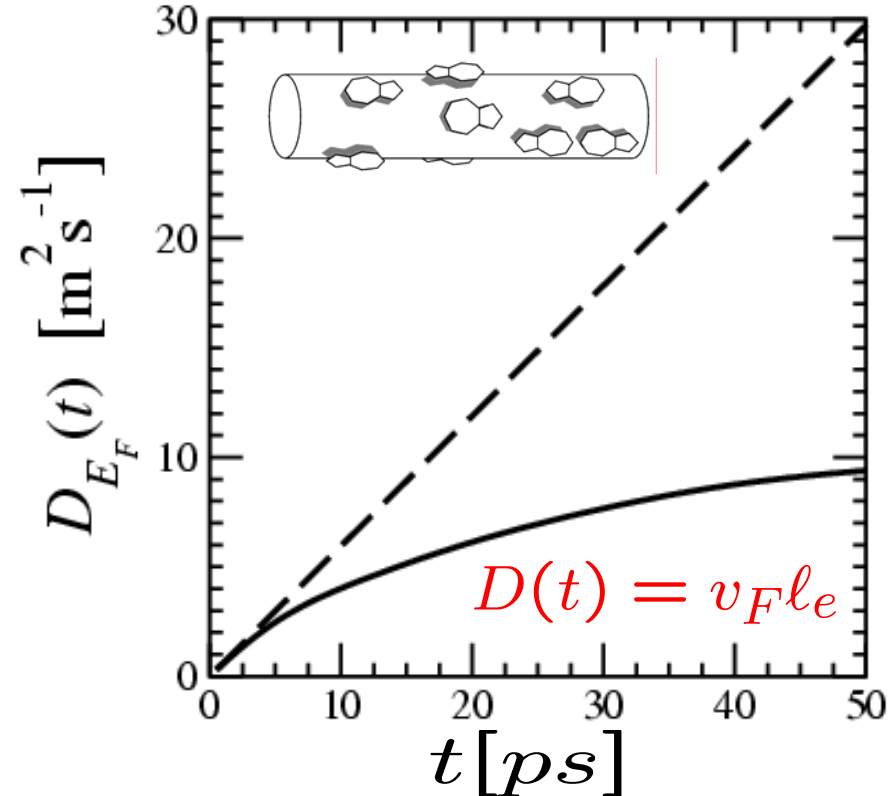
Random coverage of physisorber molecules C_6H_6 et de

$$C_{10}H_8 \quad D(E, t) = \frac{1}{t} \langle (\hat{X}(t) - \hat{X}(0))^2 \rangle_E$$

Ballistic conduction

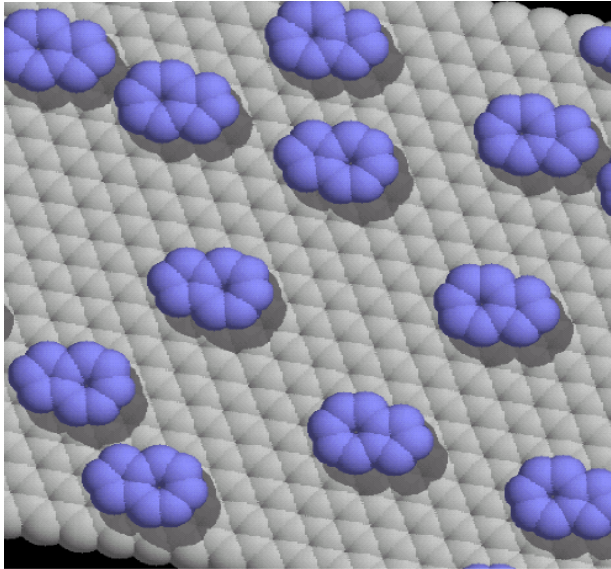
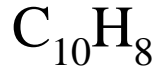


Diffusive conduction

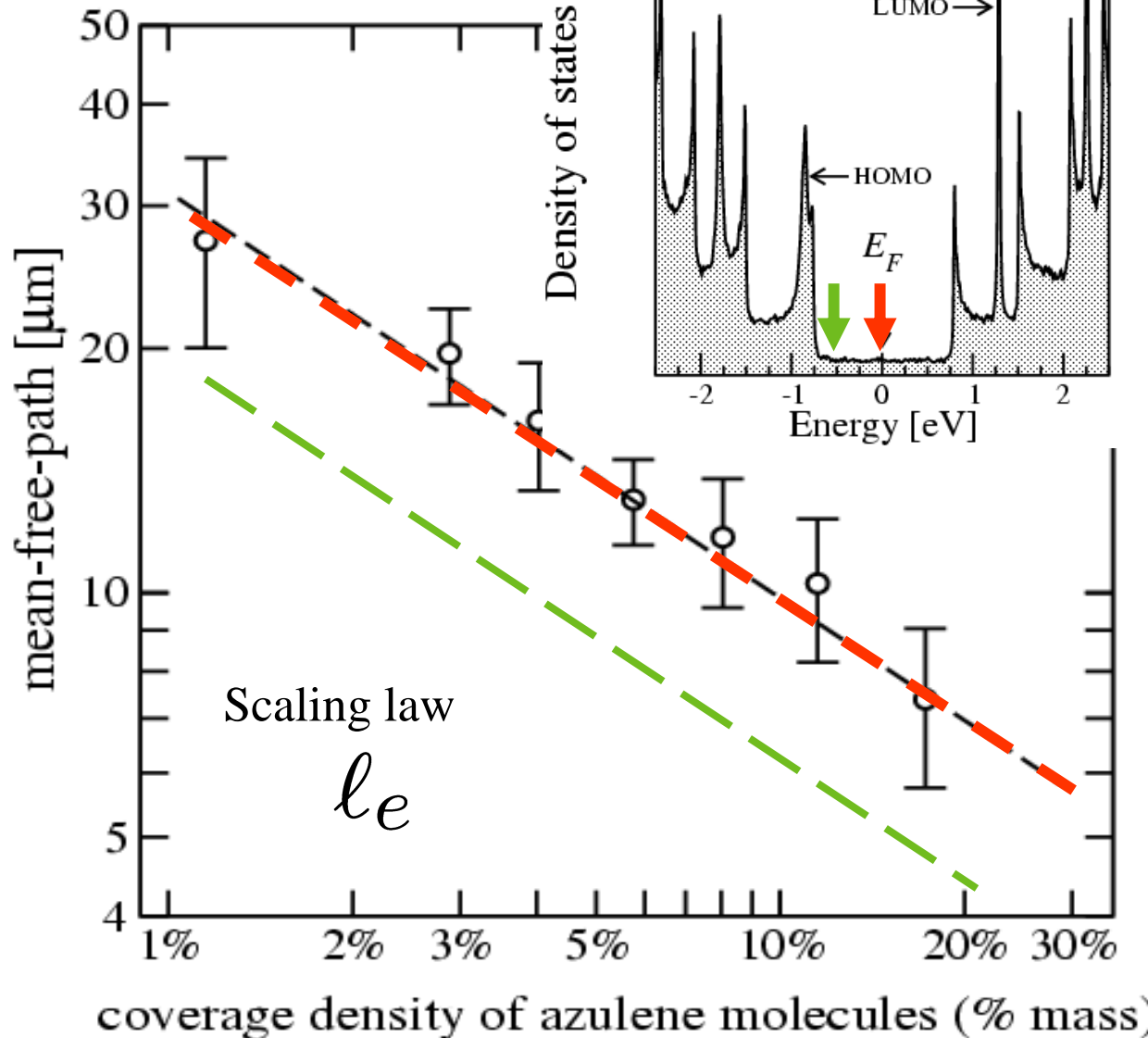


Azulene case: mean free path

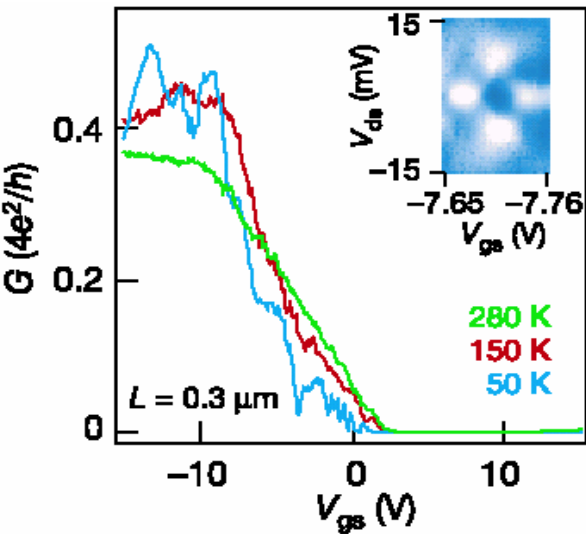
Random coverage



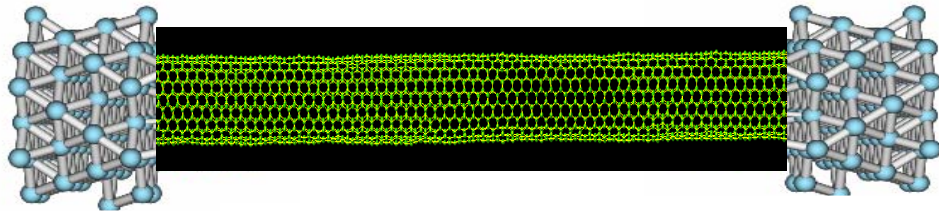
downscaling of
MFP as FL approaches
HOMO resonance

$$l_e$$


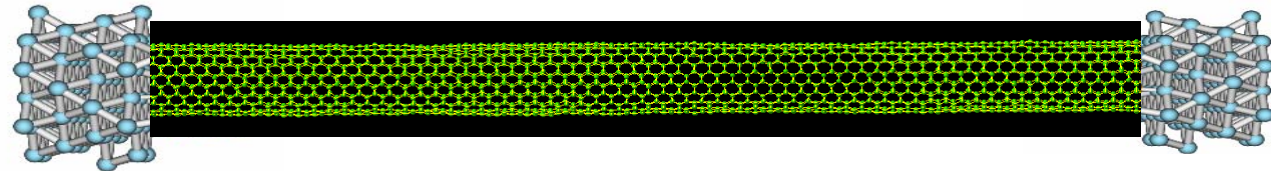
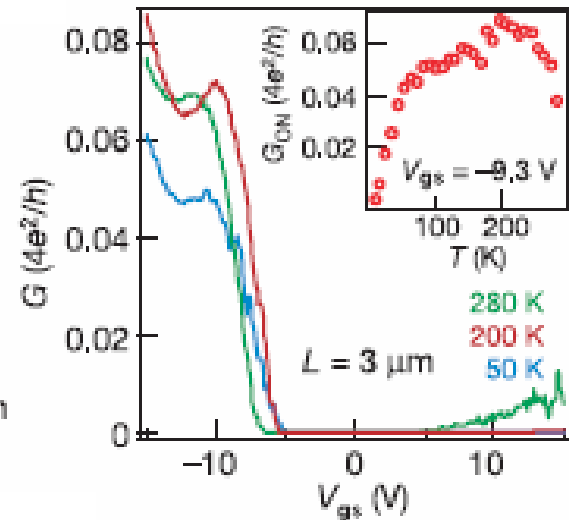
Ballistic transport limitations (high bias conditions)



Electron-phonon coupling
(high energy-optic vibrational modes)



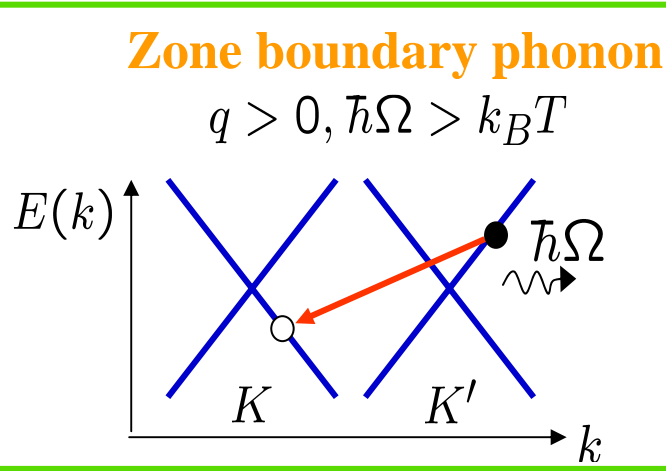
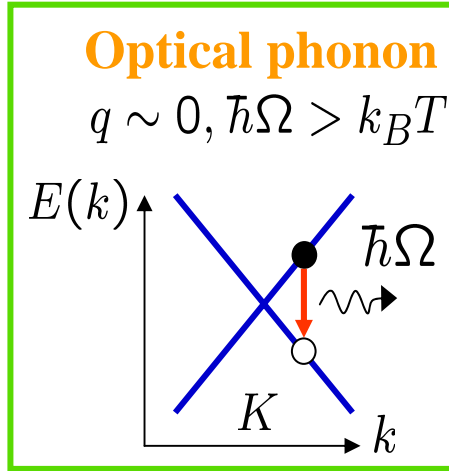
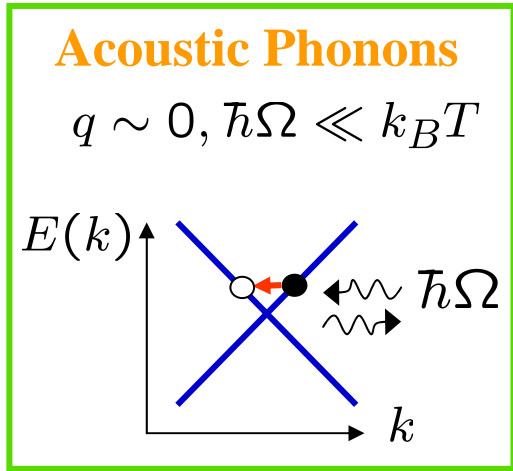
$L = 0.3 \mu\text{m} > L_{\text{el-ph}}$
Balistic regime



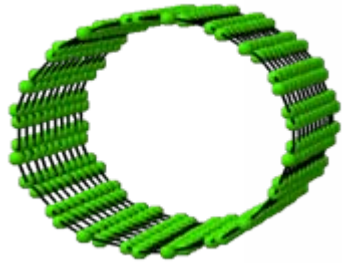
$L = 3 \mu\text{m} < L_{\text{el-ph}}$
« ohmic-like regime ? »

A. Javey, J; Guo, Q. Wang, M. Lundstrom & H.J. Dai
Nature 424, 654 (2003)

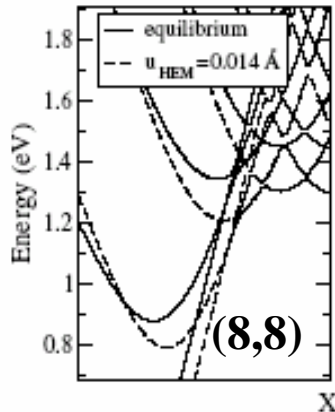
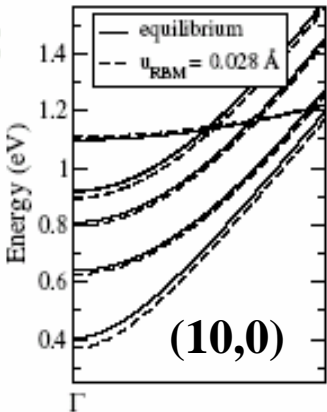
Electron-Phonon coupling in Carbon Nanotubes



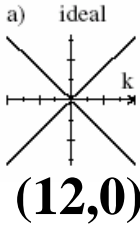
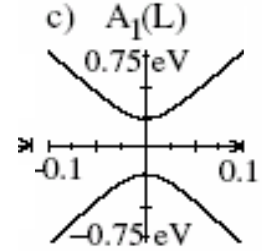
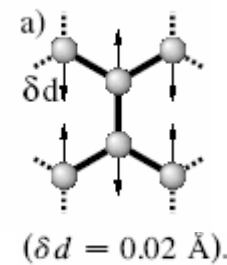
Atomic displacements & electronic structure



RBM mode : change in diameter



Optic modes $A_1(L)$



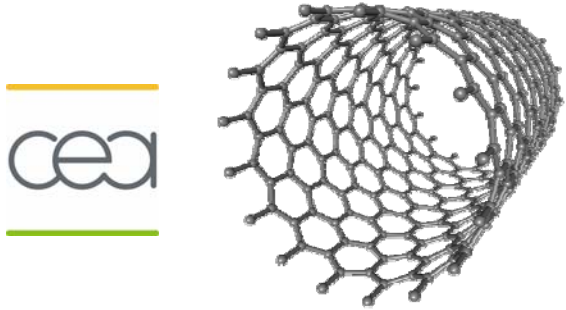
Gap opening

Ab-initio calculations

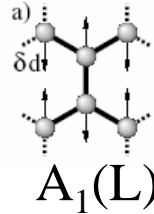
O. Dubay, G. Kresse, H. Kuzmany,
 Phys. Rev. B 88, 235506 (2002)

M. Machon, S. Reich et al., Phys. Rev. B 71, 035416 (2005)

Quantum Transport in presence of el-ph coupling



Hamiltonian



$$H_e = -\gamma_0 \sum_{\langle i,j \rangle} [c_i^\dagger c_j + h.c.],$$

$$H_{ph} = \hbar\omega_0 b^\dagger b, \quad \text{Mode LO (1580cm}^{-1}\text{)}$$

$$H_{e-ph} = \sum_{\langle i,j \rangle_{vib}} [\gamma_{i,j}^{e-ph} c_i^\dagger c_j (b^\dagger + b) + h.c.]$$

Inelastic transport computed from a many-body treatment of e-p in Fock space

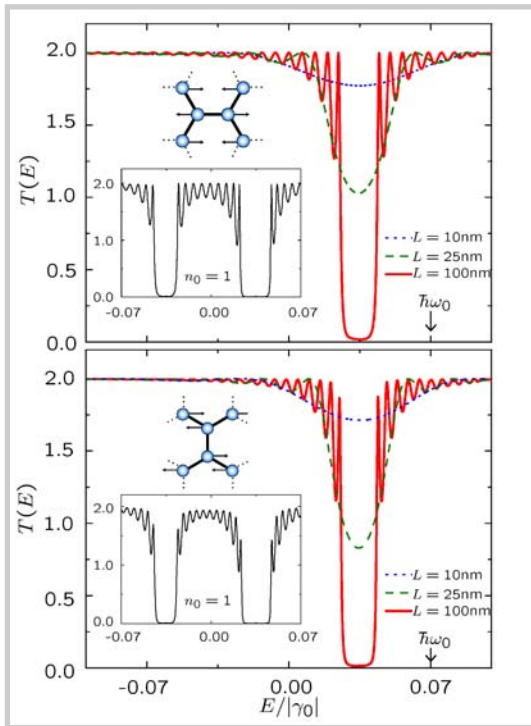
Bonca and Trugman, *PRL* 75, 2566 (1995)

Peierls-type mechanism

when the electrons gain enough energy (bias source) to emit optic phonons...

Backscattering probability = 1

Luis Foa-Torres and SR, *Phys. Rev. Lett.* 97, 076804 (2006)



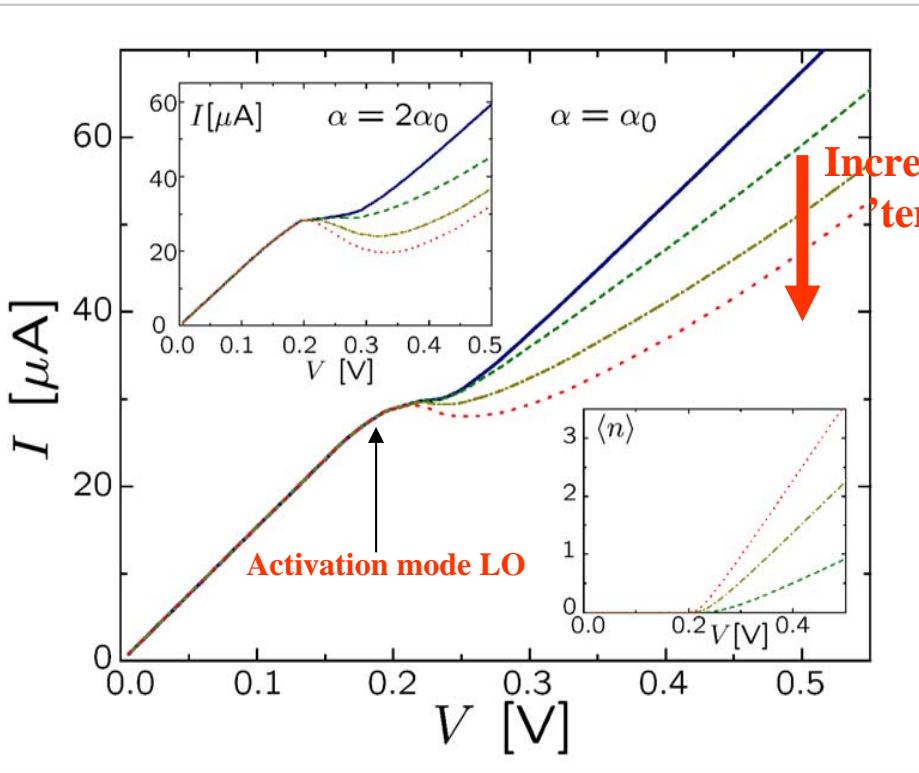
Peierls type mechanism & onset of current saturation

Ballistic regime

$$I = \frac{4e^2}{h} V$$

Onset of current saturation at $V = \hbar\omega_0$

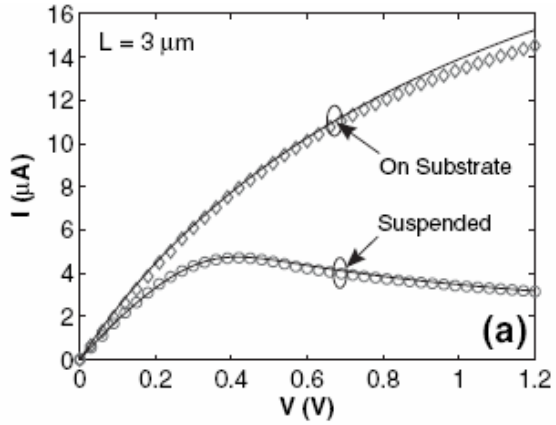
$$I_{sat} = \frac{4e}{h} \hbar\omega_0$$



Increasing phonon 'temperature'

Activation mode LO

These mechanisms are beyond the scope of a Fermi Golden rule-type description of inelastic backscattering



E. Pop et al, PRL 95, 155505 (2005)



Thank you for your attention!

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