Statistical scattering properties of disordered waveguides: closed channels contributions and the effective medium approximation

M. Yépez and J. J. Sáenz

Condensed Matter Physics Department and Centro de Investigación en Física de la Materia Condensada (IFIMAC), Universidad Autónoma de Madrid, Fco. Tomás y Valiente 7, 28049-Madrid, Spain
miztli.yepez@uam.es

Abstract.
The statistical scattering properties of disordered waveguides are studied by using a disordered system of length L, that we call the "Building Block". The Building Block is constructed as a sequence of n scattering units in the propagation direction, which are statistically independent and identically distributed. Each scattering unit is idealized as delta potential in the longitudinal direction of the waveguide, while the transverse profile is a random function.

The theoretical study is developed by using two perturbative approaches: Born series method and the transition matrix method. Both perturbative approaches consider weak scattering units and their results are obtained in the short-wave-length or weak disorder approximation, where the wave number k and the mean free path ℓ satisfy the condition $k\ell \gg 1$. The theoretical predictions are compared with numerical simulations when the waveguide supports $N = 2$ open channels (traveling modes) and $N' = 0, 1, 2, 3$ closed channels (evanescent modes) were considered in the calculations.

Born series method predicts that the closed channels contributions are crucial for the statistics of the scattering amplitudes, while the statistic of the corresponding coefficients is insensitive to those contributions. Unfortunately, this Born series method is only valid in the ballistic regime ($L \ll \ell$), where its predictions are in good agreement with the numerical simulations; however, Born series predictions suggest that the closed channels contributions are relevant for the scattering amplitudes even beyond the ballistic regime, what is confirmed by the numerical simulations: see Fig. 1.

In order to give a more general description than Born series method, a perturbative method based on the transition matrix $\mathscr{T}$ method was performed. This method explains the intriguing contributions of the closed channels in the statistics of the scattering amplitudes, considers explicitly the multiple scattering processes and gives an excellent agreement with the numerical simulations even beyond the ballistic regime: see Fig. 2. In addition, when the waveguide admits a very large number of open channels $N \gg 1$, the transition matrix method predicts that the Building Block can be replaced by an "effective potential". On the other hand, if the number of open channels is $N \sim 1$, it is not possible to approximated the Building Block by an effective medium, what is due to the contributions coming form the recurrent multiple scattering.

Keywords: Disordered waveguides; Quantum transport; Random processes
PACS: 72.80.Ng,73.23.-b,73.23.Ad,42.25.Dd

REFERENCES

[1] A. Ishimaru, *Propagation and Scattering in Random Media*, Academic Press, New York, 1978.
[2] P. Sheng, *Introduction to Wave Scattering, Localization and Mesoscopic Phenomena*, Academic Press, New York, 1995.

[3] P. A. Mello and N. Kumar, *Quantum Transport in Mesoscopic Systems. Complexity and Statistical Fluctuations*, Oxford University Press, Oxford, 2004.
[4] L. S. Froufe-Pérez, M. Yépez, P. A. Mello and J. J. Sáenz, *Phys. Rev. E*, **75**, 031113, (2007); M. Yépez, P. A. Mello and J. J. Sáenz, AIP. Conf. Proc., **1319**, 49, (2010).
[5] U. Frisch, *Annales d'Astrophysique*, **30**, 565, (1967).
[6] Ben Payne, Tom Mahler and Alexey G. Yamilov, *Waves in Random and Complex Media*, **23**, 43, (2013).

FIGURES

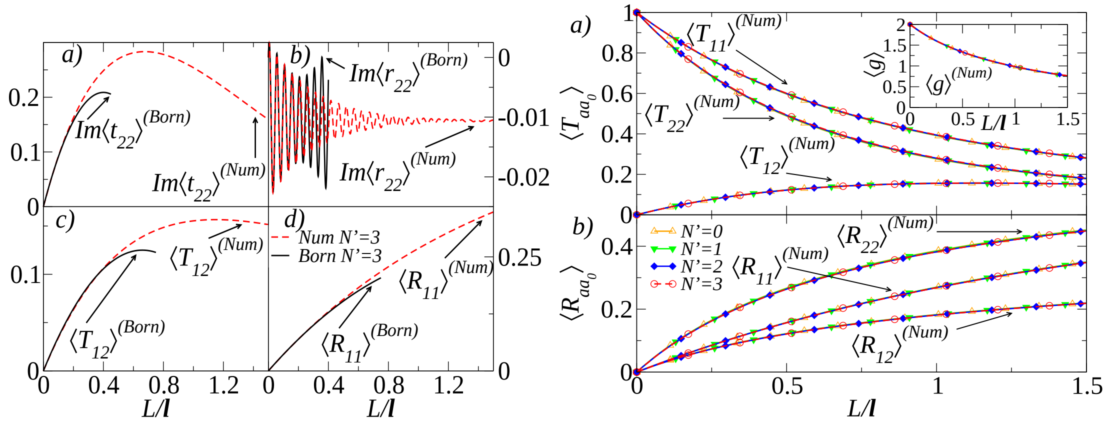

FIGURE 1. Left: Born series and numerical results for the expectation values of scattering amplitudes and coefficients when the waveguide supports $N = 2$ open channels and $N' = 3$ closed channels were considered in the calculations.

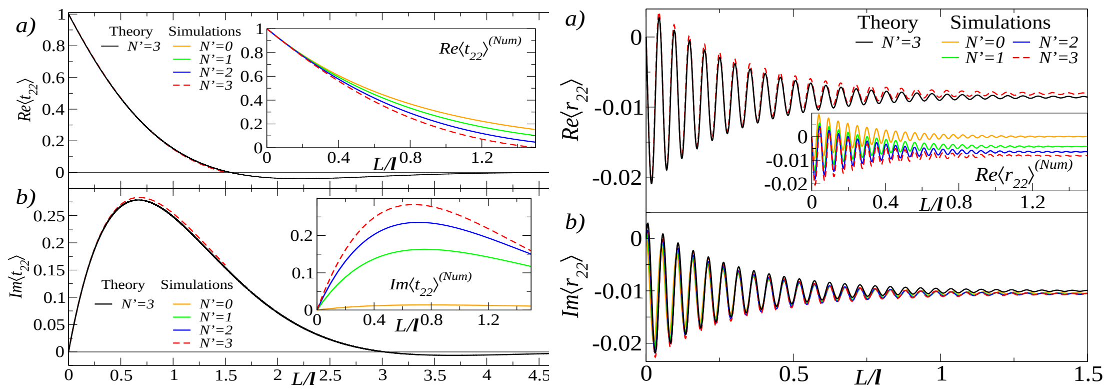

FIGURE 2. Theoretical and numerical results for the scattering amplitudes when the wave guide supports $N = 2$ open channels and $N' = 3$ closed channels were considered in the calculations.