ANALYSIS OF A SCHEME TO OBSERVE QUANTUM JUMPS OF PHONON TRNSITIONS IN A NANOMECHANICAL OSCILLATOR

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With device fabrication in the submicron or nanometer regime, it is possible to fabricate mechanical oscillators with very high fundamental frequencies and high mechanical quality factors. In the regime when the individual mechanical quanta are of the order of or greater than the thermal energy, quantum effects become important. Recently, a high-frequency mechanical resonator beam that operates at GHz frequencies has been reported [1]. For a resonator operating at the fundamental frequency of GHz and at a temperature of 100mK, on average only 20 vibrational quanta are present in the fundamental mode. An interesting question is [2]: can we observe quantum jumps, i.e., discrete (Fock or number state) transitions in such a true mechanical oscillator in a mesoscopic solid-state system, as the mechanical oscillator exchanges quanta with the outside world or environment? When achieved, experiments analogous to those of quantum optics will emerge in nanoscale solid-state phonon systems. Therefore, pursing the quantum thermal phonon transport will lead to intriguing discoveries in nanoscale physics at ultra-low energy scales.

In order to observe quantum jumps, one needs to design a scheme to measure the phonon number of the oscillator so that the oscillator will stay in a certain phonon number state long enough before it jumps to another phonon number state due to the inevitable interaction with its environment, usually through linear coupling to the oscillator position. To achieve a quantum mechanical phonon number measurement of a mechanical oscillator, conventional measurement methods, such as the direct displacement measurement, cannot be simply applied since the observable (i.e., the number of phonons in the oscillator) does not commute with, for example, the position or displacement operator. Thus, naively attaching a readout transducer to the mechanical oscillator results in inaccurate subsequent measurements due to back action. One thus must make sure that the transducer that couples to the mechanical resonator measures only the mean-square position, without coupling linearly to the resonator's position itself.

Preliminary experiments heading in this direction have been conducted at Caltech [1,2]. They use a second, driven mechanical oscillator as the transducer to measure the mean-square position of the system resonator (see Fig. 1). The basic idea is that the non-linear, quadratic-in-position coupling between the two oscillators shifts the resonance frequency of the transducer oscillator (oscillator 1) by an amount proportional to the phonon number or energy excitation of the system oscillator (oscillator 0). This frequency shift may be detected as a phase shift of the oscillations of the transducer oscillator with respect to the driving, when driven at a fixed frequency near resonance. Also, the transducer oscillator needs to have sufficient sensitivity to resolve an individual quantum jump.

We investigate this scheme of phonon number measurement using two anharmonically coupled modes of oscillation of mesoscopic elastic bridges. We analyzed the effect of higher order anharmonic terms, neglected in the previous analysis [3], in the approximation that the readout oscillator is heavily damped. In a magnetic field, a wire patterned on the moving readout oscillator will result in an induced current which can be directly monitored by electrical means. We show that the induced electromotive readout current from the readout oscillator gives direct access to the position of the readout oscillator and, through the mechanism described above, to the phonon number of the measured system oscillator, even in the presence of higher order anharmonic terms. We also obtain the relation between the current and the measured system observable. We derive the conditions for this measurement scheme to be quantum limited and find a condition on the size of the anharmonicity. If the damping of the readout oscillator is much larger than the effect of the self-anharmonic term, the overall effect of self-anharmonic term on the phonon number measurement is small.

We also derive the relation between the phase diffusion back-action noise due to number measurement and the localization time for the measured system to enter a phonon number eigenstate. We relate both these time scales to the strength of the measured signal, which is an induced current proportional to the position of the readout oscillator. We show that this measurement scheme realizes an ideal quantum non-demolition measurement of phonon number in the limit that the back action induced phase diffusion rate (or decoherence rate) , Γ, is much larger than the rate, ν, at which transitions occur between phonon number states, i.e., $(\Gamma/\nu)\to\infty$. When the ratio (Γ/ν) is finite and large, it is then possible to observe, in the readout current, quantum jumps between Fock (number states) in a mesoscopic mechanical oscillator, as the mechanical oscillator exchanges quanta with the environment. Figure 2 shows a possible trajectory of the measured system oscillator phonon number as a function of time in a single run of experiment. If we would increase the (Γ/ν) ratio in the simulations, the system oscillator would stay in a number state longer and the jumps would become more pronounced.

References:

[1] X. M. H. Huang, C. A. Zorman, M. Mehregany and M. L. Roukes, Nature, **421** (2003) 495.
[2] M.L. Roukes, Technical Digest 2000 Solid-State Sensor and Actuator Workshop (2000) [arXive: cond-mat/0008187].
[3] D. H. Santamore, A. C. Doherty, and M. C. Cross, [arXive: cond-mat/0308210].

Figures:

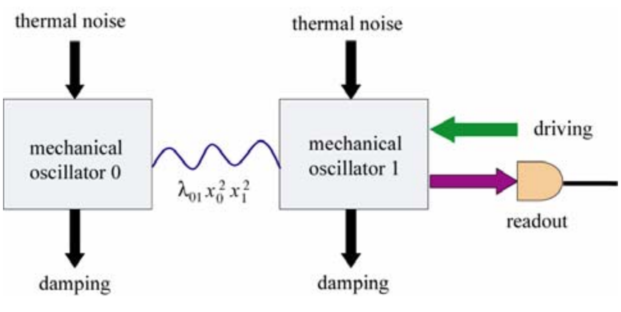

Figure 1. Schematic of the setup of a phonon number measurement for a coupled mechanical oscillator.

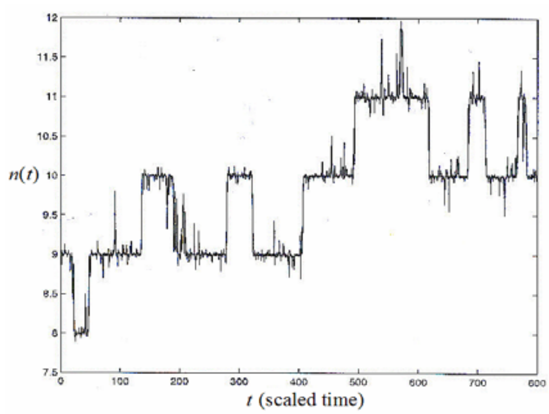

Figure 2. Simulation of a possible phonon number trajectory, *n(t)*, in a single run of experiment.