Babinet principle in optical nano-circuits and metamaterials

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Classical Babinet principle is only rigorous for infinitely thin perfect conducting screens. A different "Babinet theorem" applicable to penetrable and thick plasmonic screens is presented in this contribution.

We will consider the quasi-static 2D problem shown in Fig.1, consisting in a 2D piecewise homogeneous region filled by some media with relative dielectric constants ε_i, which supports a quasielectrostatic electric field $\mathbf{E} = -\nabla_t \phi(x, y) = -\mathbf{u}_x \partial_x \phi - \mathbf{u}_y \partial_y \phi$, where $\phi(x, y)$ satisfies Laplace's equation $\nabla_t^2 \phi = 0$. Surface plasmons and bound solutions for $\phi(x, y)$ may exist if there is at least one ε_i with $\mathrm{Re}(\varepsilon_i) < 0$, and at least one ε_j with $\mathrm{Re}(\varepsilon_j) > 0$. In such case, the electric field inside each region must satisfy $\nabla_t \times \phi = 0$ and $\nabla_t \cdot \mathrm{E}_i = 0$, as well as the boundary conditions at the border between i and j media $\mathbf{n} \times (\mathbf{E}_i - \mathbf{E}_j) = 0$ and $\mathbf{n} \cdot (\varepsilon_i \mathbf{E}_i - \varepsilon_j \mathbf{E}_j) = 0$, where n is the unit vector normal to this border and contained in the *xy* plane.

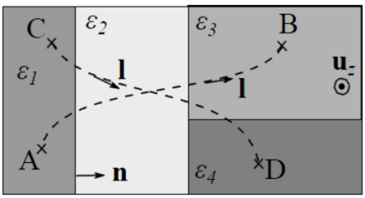

Figure 1: Illustration of a planar nanocircuit like those analyzed in this contribution.

The "complementary" structure is obtained by substituting the permittivities ε_i by the "complementary" ones $\varepsilon'_i = C_1/\varepsilon_i$ where C_1 is an arbitrary constant (for the particular case of a two-phase planar region we can choose $C_1 = \varepsilon_1\varepsilon_2$ in order to recover conventional complementarity). The "complementary" fields $\mathbf{E}'_i$ inside each region of the complementary structure are defined by:

$$\mathbf{E}'_i = C_2\, \varepsilon_i\, \mathbf{u}_z \times \mathbf{E}_i \qquad (1)$$

where C_2 is an arbitrary constant. It is shown that these fields also satisfy the quasi-electrostatic equations. Let be *A, B, C* and *D* some fixed points in the original and the complementary structures (see Fig.1). Let us define the voltage integral between *A* and *B* and the current integral through the path *C–D* as:

$$V_{AB} = \int_A^B \mathbf{E} \cdot \mathbf{l} dl; \text{ and } I_{CD} = j\omega\varepsilon_0 h \int_C^D \varepsilon(\mathbf{r}) \mathbf{E} \cdot (\mathbf{u}_z \times \mathbf{l})\, dl \quad (2)$$

where h is the thickness of the circuit board. Let us assume that we can define some meaningful impedances $Z = V_{AB}/I_{CD}$ for the structure of Fig.1 and $Z' = V'_{AB}/I'_{CD}$ for its complementary one. By using (1) in (2) we obtain: $h^2 k^2 C_1 Z Z' = -Z_0^2$ (3)

where $k = \sqrt{\varepsilon_0/\mu_0}$ is the phase constant and $Z_0 = \sqrt{\varepsilon_0/\mu_0}$ is the vacuum impedance. Let us now consider a diffraction screen made of a periodic planar nano-circuit [1]-[3]. Since nano-circuits must be electrically small [1], the periodicity must be small too. Therefore, we can describe the screen as a surface impedance sheet. This surface impedance will be, in general, a 2D symmetric tensor whose main values are related with the nano-circuit impedances. The surface impedance along a main axis of this tensor can be computed as $Z_s = Z l_{CD}/l_{AB}$ with Z defined above with the paths *A–B* and *C–D* chosen as straight lines going across the whole unit cell and directed along the proper main axes of the surface impedance tensor (l_{AB} and l_{CD} are the lengths of the corresponding paths). Specifically, this surface impedance Z_s describes the behavior of the screen for incident light polarized along the *A→B* direction. For the "complementary" screen, the surface impedance for incident light of orthogonal polarization is $Z'_s = Z' l_{CD}/l_{AB}$, with Z' defined above. The transmission coefficient for the first screen and the considered incident light is:

$$t = \frac{2Z_s}{Z_0 + 2Z_s} \qquad (4)$$

and the transmission coefficient for the complementary screen:

$$t' = \frac{2Z'_s}{Z_0 + 2Z'_s} = \frac{Z_0}{Z_0 + 2KZ_s}; \quad K =' h^2 k^2 C_1/4 \quad (5)$$

which for K = 1 reproduces the well known Babinet relation $t+t'$= 1 for infinitely thin perfect conducting complementary screens. If $K \neq 1$, Eqs. 4-5 still reproduce many of the main predictions of Babinet principle. For lossless media, the transmittance $|t|^2$ has a zero when and $|t'|^2$ has a maximum when $|Z| \rightarrow 0$. For lossy media $|Z|$ never goes to zero, and the minimum of $|t|^2$ occurs at the minimum of $|Z|$, whereas the maximum of $|t'|^2$ occurs at the minimum of $|KZ|$. This may lead to some deviation between the minimum of $|t|^2$ and the maximum of $|t'|^2$. Eqs. 4-5 can be considered as the generalization of Babinet principle for planar nano-circuits. However, they are still approximate and valid in the quasi electrostatic limit and as far as the effects of fringing fields can be neglected. Since these constrains are approximately fulfilled by many planar nano-circuit [1]-[3] and metallic metamaterial "atoms" [4]-[5], this theory is expected to be useful for the analysis of these structures.

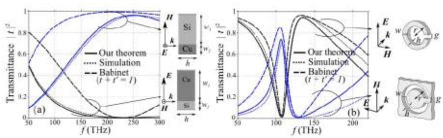

Figure 2: Left: transmission through two complementary 1D diffraction screens made of alternating layers of copper and silicon (unit cells shown aside). Dimensions are $w1$ = 50 nm, $w2$ = 10 nm and h = 25 nm. Right: transmission through complementary silver SRR and CSRR screens with r = 100 nm, g = 10 nm, w = 30 nm and h = 60 nm. Periodicity is 250 nm.

We have applied our theory to the analysis of the structure shown in the inset of Fig.2 (left). It is a 1D diffraction screen made of alternating layers of copper and silicon ($\varepsilon \approx 11.9$) which can be seen as the realization of an optical nano-circuit [6]. We first computed the transmittance through the screen using the commercial solver CST Microwave Studio, and then obtained the transmittance through the complementary screen from (4)-(5) after elimination of the common variable Z. The results are shown in Fig.2, where a very good agreement between our theory and the electromagnetic simulations can be observed. The results coming from conventional Babinet principle (t + t' = 1), also plotted, show a significant deviation from the computed ones, as expected from the properties of the media involved in the screen. Our theory has also been applied to the computation of the transmittance through screens made of conventional and complementary SRRs operating in the optical range (Fig.2 (right)), and also approaches reality better than conventional Babinet principle (t + t' = 1), in spite of the fact that SRRs can not be considered as purely quasi-electrostatic entities except at very high frequencies, i.e. beyond saturation [7]. Finally, we have computed the transverse components of the electric field displacement and the electric field in the middle plane of the SRR and the CSRR studied in Fig. 2 (right), respectively. According to our theory the orthogonal components of these fields must show a similar behavior. This fact is confirmed in Fig. 3 where the results of the aforementioned computations are shown.

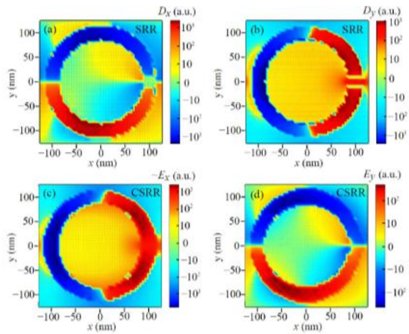

Figure 3: Electric field displacement components in the middle plane of the SRR (a), (b) and electric field components in the middle plane of the CSRR (c), (d) at a frequency of 100THz. The resemblance between the distribution of the cross components of the displacement and electric field in both structures is in accordance with the hypothesis of the theorem reflected in equation (2).

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