# Simplified coupled-wave analysis for the characterization of extraordinary optical transmission in single and stacked metallic screens

**Vicente Delgado**[1], Ricardo Marqués[1] and Lukas Jelinek[2]

[1]Dept. of Electronics and Electromagnetism, Av. Reina Mercedes S/N 41012, Univ. of Seville, Spain
[2]Dept. of Electromagnetic Field, Czech Technical Univ. in Prague 16627-Prague, Czech Republic

vdelgado @u s.es

We present a coupled-wave and surface-impedance method for the analysis of extraordinary optical transmission (EOT) through realistic metallic screens under normal and oblique incidence. Standard scattered matrix method is used to account for possible dielectric interlayers and stacked screens.

Extraordinary Optical Transmission (EOT) through metallic screens with a periodic array of subwavelength holes, first reported in [1], is a topic of intensive research with potential applications in photonic circuits [2] optical sensing [3] and fabrication of fishnet metamaterials [4]. EOT has been explained from different perspectives [5]-[7]. In our group we have proposed different waveguide models that account for EOT in the whole frequency range where it has been reported (e.g. [7], [8]). In this contribution we extend the previous work to the case of EOT screens made of realistic metals under oblique incidence. The model can also be applied to stacked and dielectric loaded of screens.

In the waveguide analysis, the problem is reduced to its unit cell (see Fig. 1) and the electromagnetic fields in the different regions are expanded in terms of Bloch and waveguide modes compatible with the boundary conditions imposed by the symmetry of the structure and the impinging wave. However, resolution of the whole problem in case of penetrable metals by these means would be inefficient. In order to overcome this difficulty, we make use of the surface impedance matrix [9] that relates the electromagnetic fields at both sides of a plain slab. If we neglect the perturbation introduced by the holes, the transverse components electromagnetic fields at both sides of the optically dense metallic screen are related through [8]

$$\begin{bmatrix} \boldsymbol{E}_{\parallel}^{(1)}(z=-t/2) \\ \boldsymbol{E}_{\parallel}^{(3)}(z=t/2) \end{bmatrix} \approx \overline{\overline{Z}} \begin{bmatrix} \hat{z}\times\boldsymbol{H}_{\parallel}^{(1)}(z=-t/2) \\ \hat{z}\times\boldsymbol{H}_{\parallel}^{(3)}(z=t/2) \end{bmatrix} \quad (1)$$

where $\overline{\overline{Z}}$ is the surface impedance matrix that can be diagonalized and is a function of the constitutive parameters of the material composing the screen and its tickness. Eq. (1) together with continuity of the fields in the cross section of the holes

$$\begin{bmatrix} \boldsymbol{E}_{\parallel}^{(1)}(z=-t/2) \\ \boldsymbol{H}_{\parallel}^{(1)}(z=-t/2) \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_{\parallel}^{(2)}(z=-t/2) \\ \boldsymbol{H}_{\parallel}^{(2)}(z=-t/2) \end{bmatrix} \text{ and } \begin{bmatrix} \boldsymbol{E}_{\parallel}^{(2)}(z=t/2) \\ \boldsymbol{H}_{\parallel}^{(2)}(z=t/2) \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_{\parallel}^{(3)}(z=t/2) \\ \boldsymbol{H}_{\parallel}^{(3)}(z=t/2) \end{bmatrix} \quad (2)$$

provide a complete set of equations for the amplitudes of the different modes in which the fields are expanded after applying integral boundary conditions.

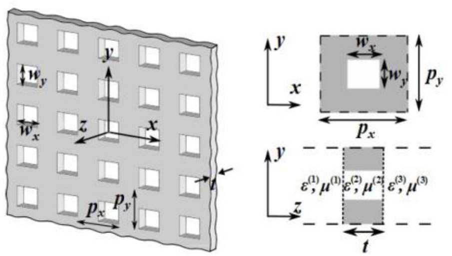


**Figure 1**: Metallic screen with a periodic array of holes (EOT screen) and fron and lateral views of the unit cell of the structure.

In Fig. 2 the transmission coefficients through a silver screen at infrared frequencies, for different polarizations and angles of incidence, obtained with the reported model are compared with electromagnetic simulations using *CST Microwave Studio*. In order to obtain meaningful results, the resolution of the highest mode inside the holes must be similar to that of the input and output regions. We employed higher order of 2 for modes inside the holes and 8 for modes in the cross section of the unit cell, which were enough to obtain accurate results compared with full wave electromagnetic simulations. CPU time per frequency point was $\sim 0.5$ s vs $\sim 4$ min with the electromagnetic solver.

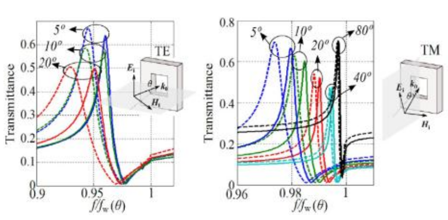


**Figure 2:** Transmission through an array of square holes in a silver screen at oblique incidence. Square periodicity is 1µm. Size of the holes is 250 nm and thickness of the screen is 50 nm. Wood's frequencies (fw) range from 150 THz to 220 THz. The metal is modeled by a Drude Lorentz permittivity. Solid lines correspond to the mode matching model and dashed lines to CST simulations.

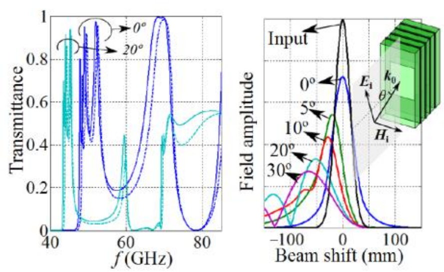


**Figure 3:** (Left) Transmission through four stacked copper screens at oblique incidence (TM incident wave). The four metallic screens are placed among a total of five dielectric slabs. Periodicities are 1.5 and 3.4 mm, size of the square holes is 1.1 mm, thickness of the metallic layer is 35 µm and thickness of each of the dielectric slabs is 0.49 mm. Continuous lines correspond to the mode matching model and dashed lines to the CST simulations. (Right) Electric field profiles of a 1D Gaussian beam impinging at different angles of incidence at the frequency of the maximum transmission. Deviation of the beam to the left is higher for increasing angles of incidence.

In case of a dielectric loaded or stacks with a finite number screens, the scattering matrix method is used in order to compute the complete scattering matrix (note that cascading of transmission matrix is ill conditioned due to the presence of positive and negative exponentials).

In Fig. 3 (left) we show the transmission coefficient for TM plane waves impinging at several angles over dielectric loaded stacked EOT screens, with an structure similar to those in [10]. Once the transmission coefficients for incident plane waves are known, we can analyze the behavior of incident Gaussian beams, which can be decomposed into plane waves. The profiles of the input and output TM beams impinging at different angles are shown in Fig. 3 (right) confirming the existence of negative refraction in accordance to the experimental results in [10].

## References


[1] T. W. Ebbesen et al., Nature, vol. 391, pp. 667–669, 1998.

[2] E. Deveux et al., Appl. Phys. Lett., vol. 83, pp. 4396–4398, 2003.

[3] A. A. Yanik et al., Nano Lett., vol. 10, pp. 4962–4969, 2010.

[4] M. Beruete et al., Opt. Express, vol. 14, pp. 5445–5455, 2006.

[5] H. F. Ghaemi, et al. Phys. Rev. B, vol. 58, pp. 6779–6782, 1998.

[6] L. Martín-Moreno, et al. Appl. Phys. Lett., vol. 86, pp. 1114–1117, 2001.

[7] F. Medina, et al. IEEE Trans. Microw. Theory Tech., vol. 56, pp. 3108–3120, 2008.

[8] V. Delgado, et al. Opt. Express, vol. 18, pp. 6506–6515, 2010.

[9] S. Tretyakov, Analytical Modeling in Applied Electromagnetics, Edt. Artech House, 2003.

[10] M. Beruete, et al. Phys. Rev. B, vol. 79, pp. 195107(1)–195107(6), 2009.