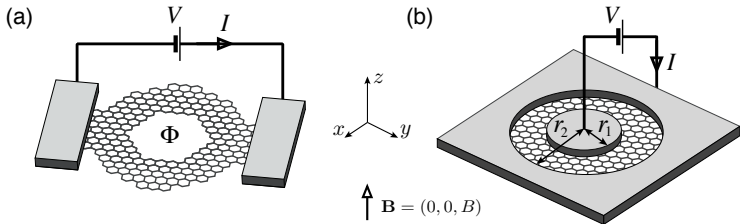# Nonstandard quantum interference graphene rings and disks

Adam Rycerz

Marian Smoluchowski Institute of Physics, Jagiellonian University, Kraków, Poland

Montréal, Canada, October 14-16, 2015

Introduction   **Emergent fermions in graphene**
Graphene rings   Valleytronics in graphene
Graphene disks   Breaking the valley degeneracy

Emergent Dirac fermions in graphene

4 August 1972, Volume 177, Number 4047

SCIENCE

More Is Different

Broken symmetry and the nature of
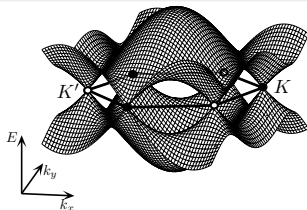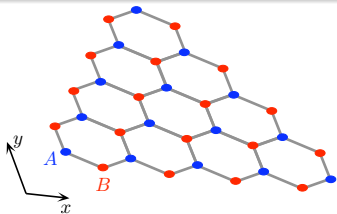the hierarchical structure of science.

P. W. Anderson

The reductionist hypothesis may still be a topic for controversy among philosophers, but among the great majority of active scientists I think it is accepted

planation of phenomena in terms of known fundamental laws. As always, distinctions of this kind are not unambiguous, but they are clear in most cases. Solid state physics, plasma physics, and perhaps

What differs graphene from a collection of carbon atoms?

- The **valley pseusospin** ($\Rightarrow$ *fermion doubling*)
- Time-reversal symmetry breaking at zero magnetic field
- Pseudodiffusive charge transport and more ...

Introduction   **Emergent fermions in graphene**
Graphene rings   Valleytronics in graphene
Graphene disks   Breaking the valley degeneracy

Emergent Dirac fermions and valleys

Tight-binding Hamiltonian:

$$\mathcal{H}_{\text{TBA}} = \sum_{\langle ij \rangle} \left[t_{ij}(\mathbf{A}) |i\rangle\langle j| + \text{h.c.} \right],$$

$$t_{ij}(\mathbf{A}) = -t \exp\left[i \frac{2\pi}{\Phi_0} \int_{\mathbf{r}_i}^{\mathbf{r}_j} \mathbf{A} \cdot d\mathbf{r} \right],$$

$t \approx 3\,\text{eV}$, and $\Phi_0 = h/e$.

Envelope wavefunction:

$$\Psi(\mathbf{r}) = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} e^{i\mathbf{K}\cdot\mathbf{r}} + \begin{pmatrix} \psi'_A \\ \psi'_B \end{pmatrix} e^{i\mathbf{K}'\cdot\mathbf{r}}.$$
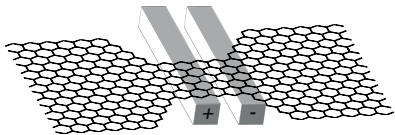
For K-point and $\mathbf{A} = 0$:

$$\frac{\hbar v_F}{i} \begin{pmatrix} 0 & \partial_x - i\partial_y \\ \partial_x + i\partial_y & 0 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = E \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix},$$
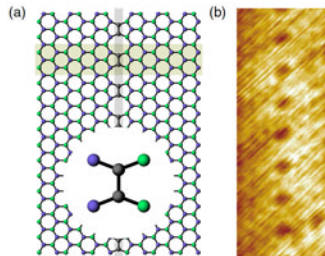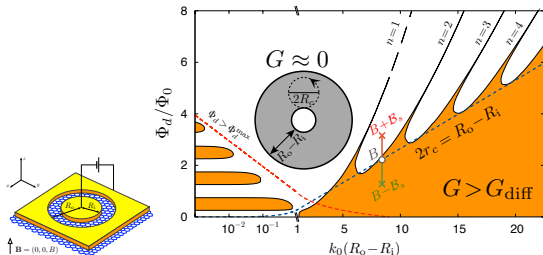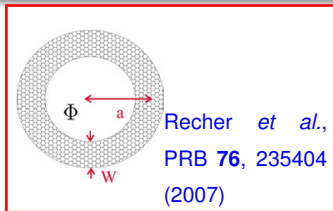
with $v_F \equiv \frac{\sqrt{3}}{2} ta/\hbar \approx 10^6$ m/s, $a = 0.246$ nm.

A compact form: $H_0\Psi = E\Psi$, with $H_0 = v_F \boldsymbol{\sigma} \cdot \boldsymbol{p}$, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$, and $\boldsymbol{p} = -i\hbar(\partial_x, \partial_y)$. For $B \neq 0$: $\boldsymbol{p} \to \boldsymbol{p} - \frac{e}{c}\boldsymbol{A}$.

Introduction   Emergent fermions in graphene
Graphene rings   **Valleytronics in graphene**
Graphene disks   Breaking the valley degeneracy

Valleytronics in graphene

AR, Tworzydło, Beenakker, Nat. Phys. (2007)

Recher *et al.*, PRB **76**, 235404 (2007)

AR, PRB **81**, 121404(R) (2010);
Khatibi *et al.*, PRB **88**, 195426 (2013).
Related: Fujita et al., APL **97**, 043508 (2010)

Gunlycke and White, PRL **106**, 136806 (2011)

Introduction   Emergent fermions in graphene
Graphene rings   Valleytronics in graphene
Graphene disks   **Breaking the valley degeneracy**

Breaking the valley degeneracy

- Lowest mode in constriction w/zigzag edges, domain walls in BLG, etc. [⇒ *not this talk ...*]
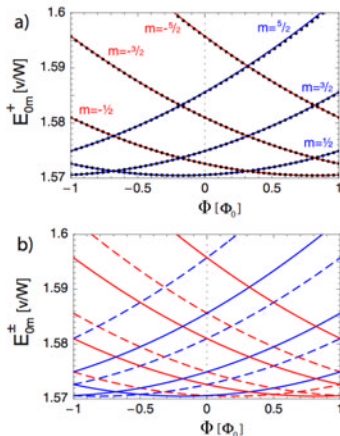
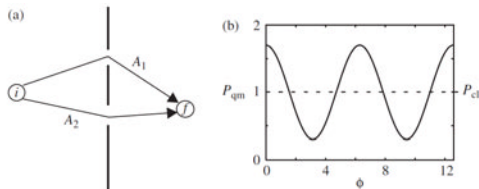- Aharonov-Bohm rings [⇒ Recher, Trauzettel, AR *et al.*, PRB **76**, 235404 (2007)]
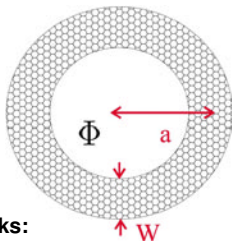
Related works:

Xu *et al.*, Sci. Rep. **5**, 8963 (2015); Culcer *et al.*, PRL **108** 126804 (2012); Pályi and Burkard, PRL **106** 086801 (2011); Recher *et al.*, PRB **79**, 085407 (2009); ...

Introduction
Graphene rings
Graphene disks

The condensed-matter two-slit experiment
Aharonov-Bohm effect in graphene rings
A-B effect and valley polarization

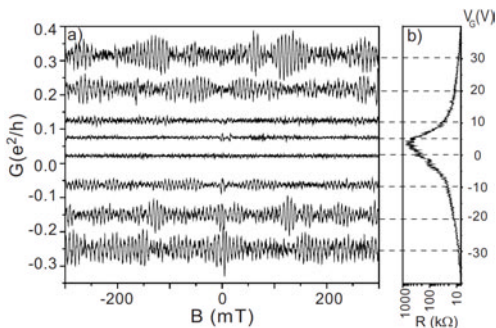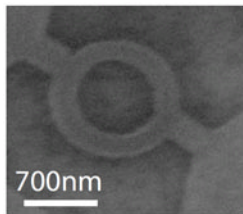# The condensed-matter two-slit experiment

Two-slit experiment. Quantum interference between two trajectories (a) results in oscillatory dependence of the propagation probability on the phase shift between two amplitudes ((b), plotted for $P_1/P_2 = 6$).

$\Rightarrow$ In metallic (or semiconducting) Aharonov-Bohm rings one cannot neglect *looped* trajectories. This may effect the oscillation period [Sharvin and Sharvin, JETP Lett. **34**, 272 (1981); Webb et al., PRL **54**, 2696 (1985).]

$\Rightarrow$ Fourier analysis necessary to determine the character of interference in a given A-B ring.

[Image from: Nazarov and Blanter, *Quantum Transport: Introduction to Nanoscience,* Cambridge University Press (Cambridge, 2009).]

Introduction	The condensed-matter two-slit experiment
Graphene rings	**Aharonov-Bohm effect in graphene rings**
Graphene disks	A-B effect and valley polarization

Aharonov-Bohm effect in graphene rings

$\Rightarrow$ In graphene A-B rings, experiments show oscillations with the standard $\Phi_0 = h/e$ period, and the magnitude $\Delta G \propto G$ (and also $\Delta G \propto T^{-1/2}$), indicating the *tunneling* transport regime [Russo et al., PRB **77**, 085413 (2008); Stampfer et al., Int. J. Mod. Phys. **23**, 2647 (2009).]

Introduction
Graphene rings
Graphene disks

The condensed-matter two-slit experiment
Aharonov-Bohm effect in graphene rings
**A-B effect and valley polarization**

A-B effect and valley polarization
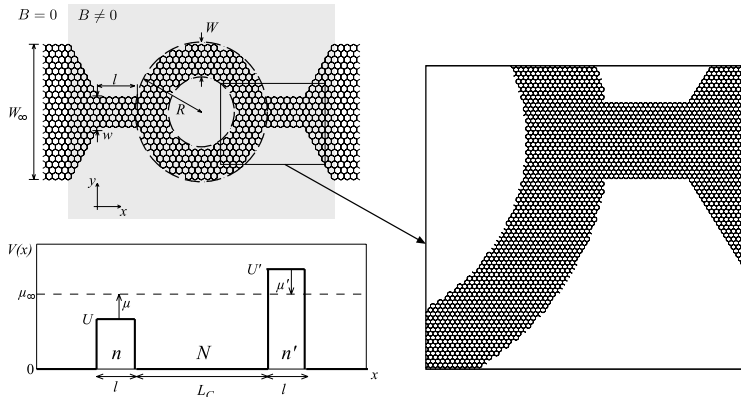
AR, Acta Phys. Polon. A **115**, 322 (2009).

Related works: Zarenia *et al.*, PRB **81**, 045431 (2010); Wurm *et al.*, Semicond. Sci. Technol. **25**, 034003 (2010); Schelter *et al.*, PRB **81**, 195441 (2010).

Introduction   **The condensed-matter two-slit experiment**
Graphene rings   Aharonov-Bohm effect in graphene rings
Graphene disks   **A-B effect and valley polarization**

A-B effect and valley polarization
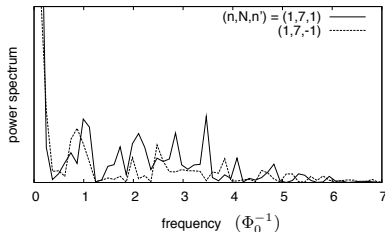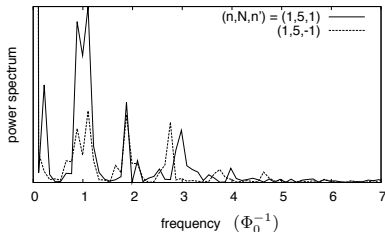
[AR, Acta Phys. Polon. A **115**, 322 (2009)]

$\Rightarrow$ Tunneling regime ($\Delta G \propto G$) reproduced in simulations when disorder taken into account [Wurm et al. (2010)].

$\Rightarrow$ *Frequency doubling* upon inversion of valley polarity (in one constriction only) not yet observed.

Introduction
Graphene rings
Graphene disks

The Corbino disk in graphene
Trigonal warping in bilayer graphene
Magnetotransport in BLG disks

The Corbino disk in graphene
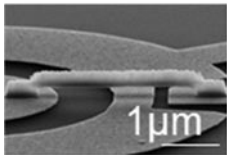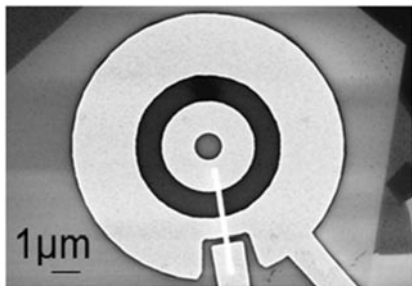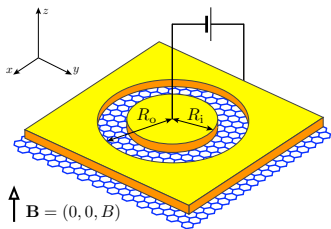
Bottom left: AR, PRB **81**, 121404(R) (2010). *Right:* Peters et al., APL **104**, 203109 (2014).

Introduction   **The Corbino disk in graphene**
Graphene rings   Trigonal warping in bilayer graphene
Graphene disks   Magnetotransport in BLG disks

Magnetoconductance of the Corbino disk in MLG

- Periodic, approximately sinusoidal, conductance oscillations
  [with the period $\Phi_0 = 2(h/e)\mathcal{L}$, $\mathcal{L} = \ln(R_o/R_i)$] appear if
  $|\Phi_d| \lesssim \Phi_d^{\max} = -(2h/e)\ln(k_0 R_i)$, where $\Phi_d = \pi B(R_o^2 - R_i^2)$ and
  $k_0 = |E_F|/(\hbar v_F)$. / $\Rightarrow$Similarly – for higher LLs. /

- Conductance for the Dirac point ($k_0 \to 0$) reads
  $G = G_{\mathrm{diff}} + \sum_{m=1}^{\infty} G_m \cos(2\pi m \Phi_d/\Phi_0)$, where $G_{\mathrm{diff}} = 2g_0/\mathcal{L}$,
  $G_m = 4\pi^2(-)^m mg_0/(\mathcal{L}^2 \sinh(\pi^2 m/\mathcal{L}))$, and $g_0 = 4e^2/h$.

A.Rycerz   **Nonstandard quantum interference**

Introduction
Graphene rings
Graphene disks

The Corbino disk in graphene
Trigonal warping in bilayer graphene
Magnetotransport in BLG disks

Trigonal warping in bilayer graphene

- Landauer conductivity [Snyman & Beenakker, 2007], $\gamma_3 = \gamma_4 = 0$:

$$\sigma_{\text{bilayer}} = GL/W = 2\sigma_0$$

- Kubo conductivity [Cserti et al., 2007], $\gamma_3 \neq 0$:

$$\sigma_{\text{bilayer}} = 6\sigma_0 \ (!)$$

- *Experiment:* [Mayorov et al., 2011] $\sigma_{\text{bilayer}} \simeq 5\sigma_0$.

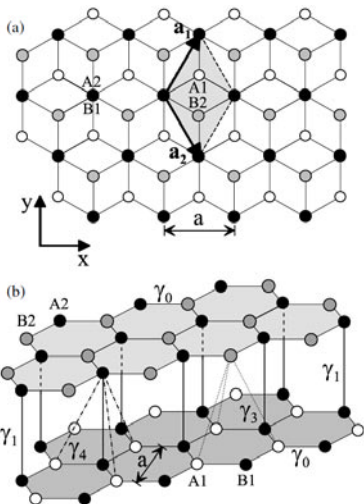[$\sigma_0 = (4/\pi)e^2/h$ – *universal conductivity of a monolayer.*]

Table 1. Values (in eV) of the SWM model parameters [64–67] determined experimentally. Numbers in parentheses indicate estimated accuracy of the final digit(s). The energy difference between dimer and non-dimer sites in the bilayer is $\Delta' = \Delta - \gamma_2 + \gamma_5$. Note that next-nearest layer parameters γ_2 and γ_5 are not present in bilayer graphene.

Parameter	Graphite [67]	Bilayer [76]	Bilayer [55]	Bilayer [56]	Bilayer [80]	Trilayer [82]
γ_0	3.16(5)	2.9	3.0[a]	—	3.16(3)	3.1[a]
γ_1	0.39(1)	0.30	0.40(1)	0.404(10)	0.381(3)	0.39[a]
γ_2	−0.020(2)	—	—	—	—	−0.028(4)
γ_3	0.315(15)	0.10	0.3[a]	—	0.38(6)	0.315[a]
γ_4	0.044(24)	0.12	0.15(4)	—	0.14(3)	0.041(10)
γ_5	0.038(5)	—	—	—	—	0.05(2)
Δ	−0.008(2)	—	0.018(3)	0.018(2)	0.022(3)	−0.03(2)
Δ'	0.050(6)	—	0.018(3)	0.018(2)	0.022(3)	0.046(10)

[a] This parameter was not determined by the given experiment, the value quoted was taken from previous literature.

[55] Zhang *et al.*, Phys. Rev. B **78**, 235408 (2008).

[56] Li *et al.*, Phys. Rev. Lett. **102**, 037403 (2009).

[67] Dresselhaus & Dresselhaus, Adv. Phys. **51**, 1 (2002).

[76] Malard *et al.*, Phys. Rev. B **76**, 201401(R) (2007).

[80] Kuzmenko *et al.*, Phys. Rev. B **80**, 165406 (2009).

[82] Taychatanapat *et al.*, Nature Phys. **7**, 621 (2011).

⇒ Novel phenomena governed solely by γ_3 VERY desired!

Introduction
Graphene rings
Graphene disks

The Corbino disk in graphene
Trigonal warping in bilayer graphene
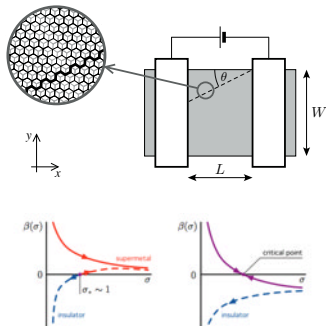Magnetotransport in BLG disks

Fig. 1: (Colour on-line) Schematic scaling functions $\beta(\sigma)$ (3) for two-dimensional disordered Dirac (solid lines) and spin-orbit (dashed lined) systems. Left: noninteracting case [17]; right: Coulomb interaction included [19]. Arrows indicate the flows of the dimensionless conductivity σ with increasing L.

Grzegorz Rut & AR:
⇒ PRB **89**, 045421 (2014)
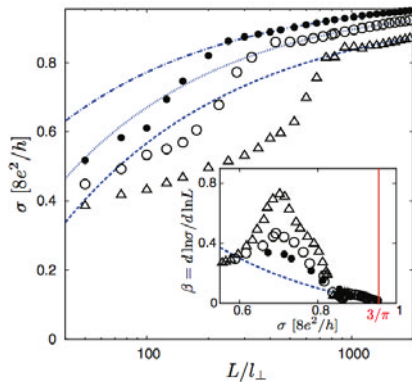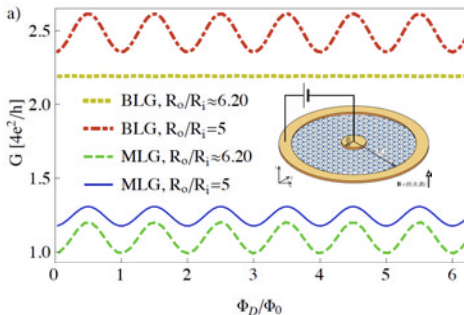⇒ EPL **107**, 47005 (2014)

[Notation: $t' \equiv \gamma_3$]

Fig. 3: (Colour on-line) Minimal conductivity of an unbiased graphene bilayer as a function of the sample length L (specified in units of $l_\perp = \hbar v_F/t_\perp \simeq 1.60$ nm). Different datapoints correspond to different values of the next-nearest neighbor interlayer hopping: $t' = 0.1$ eV ($\triangle$), 0.2 eV ($\bigcirc$), and 0.3 eV ($\bullet$).

Introduction
Graphene rings
Graphene disks

The Corbino disk in graphene
Trigonal warping in bilayer graphene
Magnetotransport in BLG disks

Magnetoconductance of BLG disk: t'=0
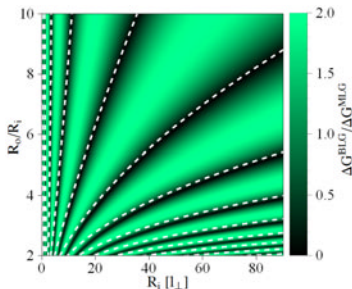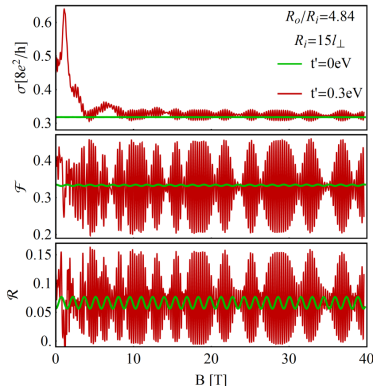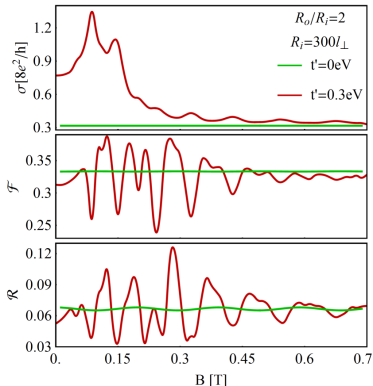

$$\Phi_D = \pi B(R_o^2 - R_i^2),$$
$$\Phi_0 = 2(h/e)\ln(R_o/R_i),$$
$$G_{\text{diff}}^{BLG} = 2G_{\text{diff}}^{MLG} = \frac{8e^2}{h}\frac{1}{\ln(R_o/R_i)}.$$

$\Rightarrow$ Oscillations magnitude at the Dirac point:

$0 \leqslant \Delta G^{\mathrm{BLG}} \leqslant 2\Delta G^{\mathrm{MLG}}$.

$\Rightarrow$ The oscillations vanish ($\Delta G^{\mathrm{BLG}} = 0$) for
$R_o/R_i \simeq [R_i t_\perp/(2\hbar v_F)]^{4/p}$,
with $p = 1, 3, 5, \ldots$.

A.Rycerz   **Nonstandard quantum interference**

Introduction
Graphene rings
Graphene disks

The Corbino disk in graphene
Trigonal warping in bilayer graphene
Magnetotransport in BLG disks

Magnetotransport in BLG disk: t'≠0

⇒ The *excess conductance decays* as $G_{\text{diff}}(t') - G_{\text{diff}}(0) \propto 1/B$ above the crossover field $B_\star \approx 2\hbar t'/(eL)$.

⇒ However, the *beats* remain, with $T_{\text{beat}} \propto \sqrt{B}$.

[Grzegorz Rut & AR, unpublished.]

Introduction
Graphene rings
Graphene disks

The Corbino disk in graphene
Trigonal warping in bilayer graphene
Magnetotransport in BLG disks

Conclusions & Acknowledgments

- Both the conductivity scaling with a sample size and the single-device magnetoconductance spectrum of BLG Corbino disk may allow one to determine skew-interlayer hopping γ_3.

- A 'common wisdom' saying that *the trigonal warping has no effect starting from few-Tesla fields* is put in question.

Collaboration:

Grzegorz Rut (PhD student)

Funding:

Project web site: http://th.if.uj.edu.pl/~adamr/sonata.html