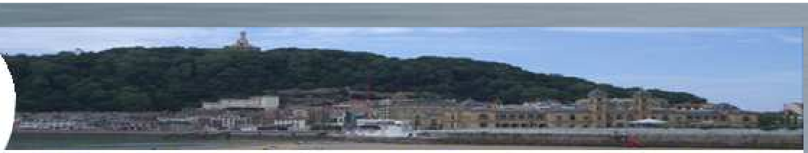
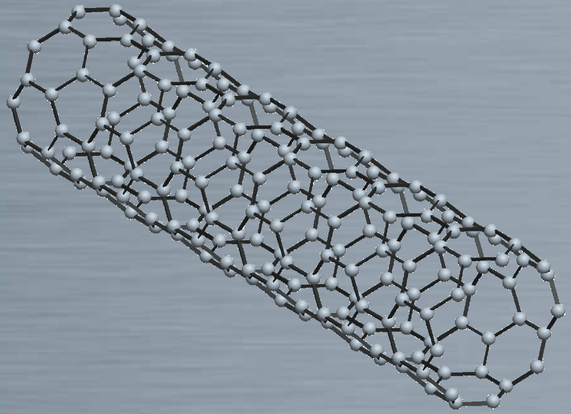
nanoICT coordination action



NanoICT School 2009 San Sebastian (Spain): October 26-30, 2009

Applying Magnetic Fields to Carbon-based low Dimensional Materials: from Aharonov-Bohm effects to Landau levels



Stephan Roche

cea

INAC
INSTITUT NANOSCIENCES
ET CRYOGÉNIE

LSim

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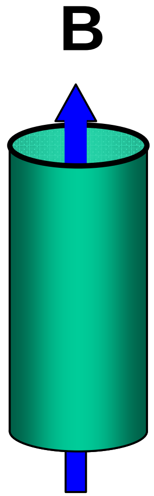
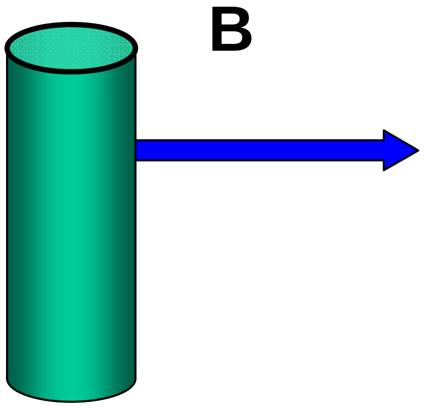
CIN2
CENTRE D'INVESTIGACIÓ
EN NANOCIÈNCIA
I NANOTECNOLOGIA
CAMPUS UAB. BELLATERRA. BARCELONA

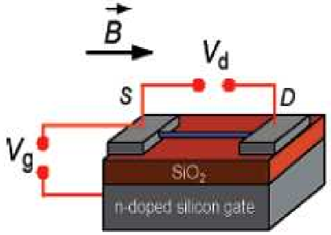
High Magnetic Field Effects in CNTs



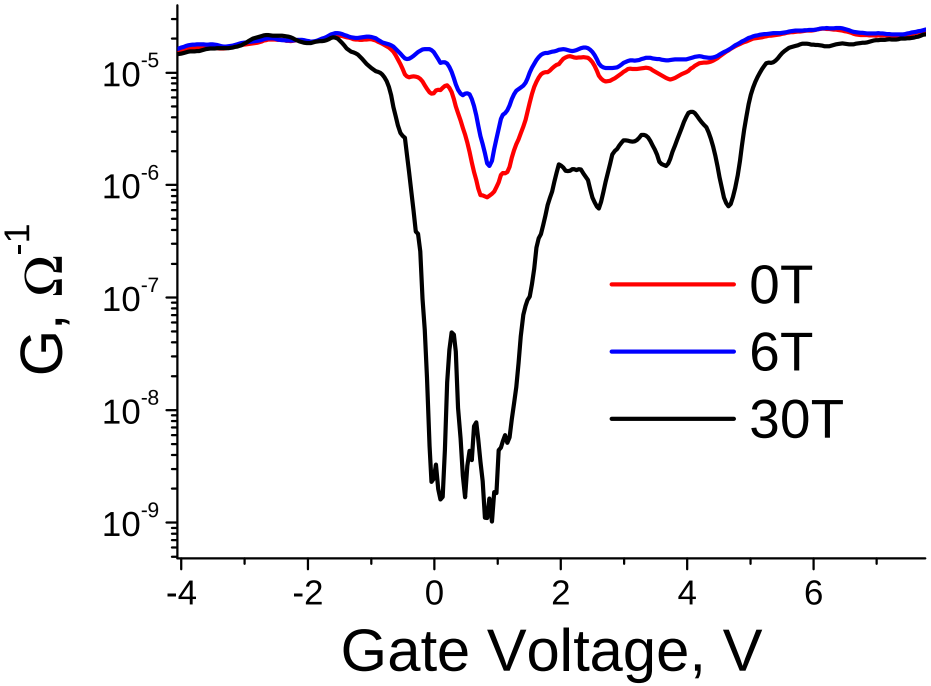
MAGNET LAB
NATIONAL HIGH MAGNETIC FIELD LABORATORY
FLORIDA STATE UNIVERSITY - LOS ALAMOS NATIONAL LABORATORY - UNIVERSITY OF FLORIDA

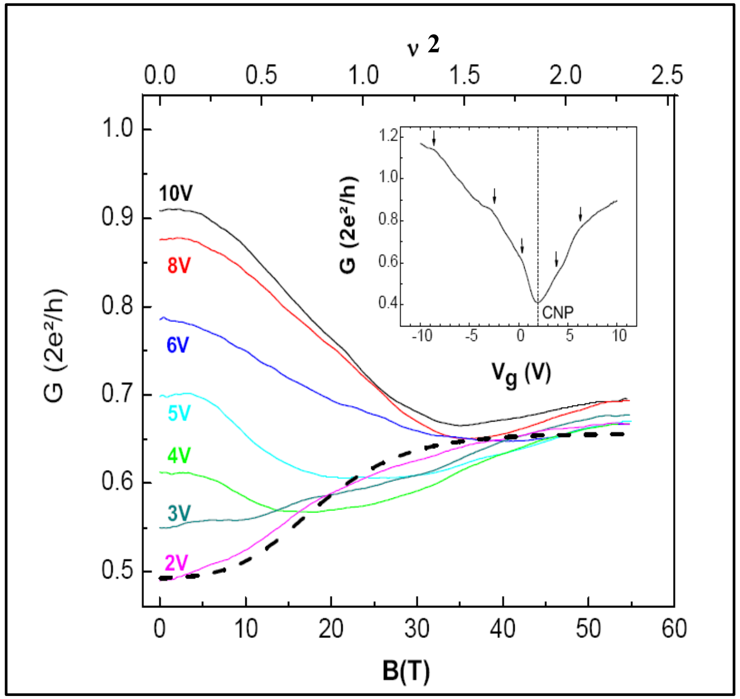
NHMFL,Tallahassee, Florida,
“RRC Kurchatov Institute”, Moscow





LNCMI



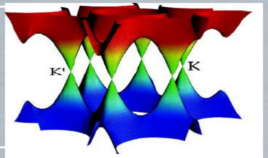


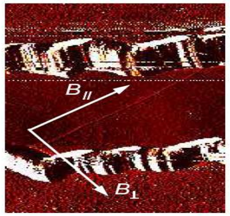
OUTLINE of the TALK

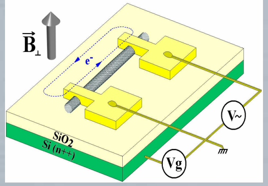
1. **INTRODUCTION**
   Basics of sp^2 electronic features

2. **Aharonov-Bohm Effect in CNTs**
   Theory and original controversies
   Magnetic field induced metal-semiconductor transition

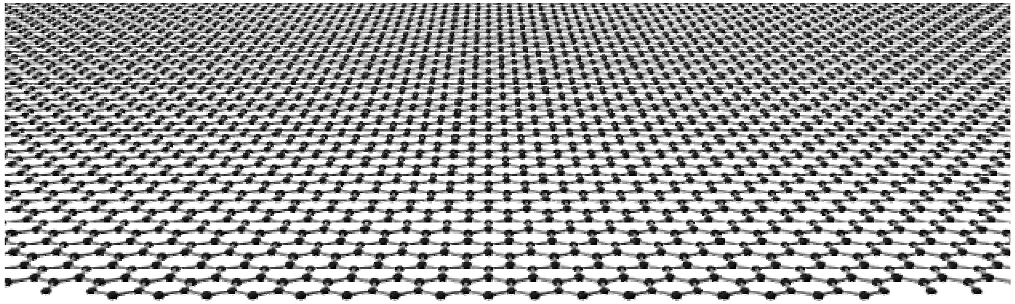
3. **Landau levels in CNTs**
   Fabry-Perot regime
   Propagative Landau levels and Fermi level pinning

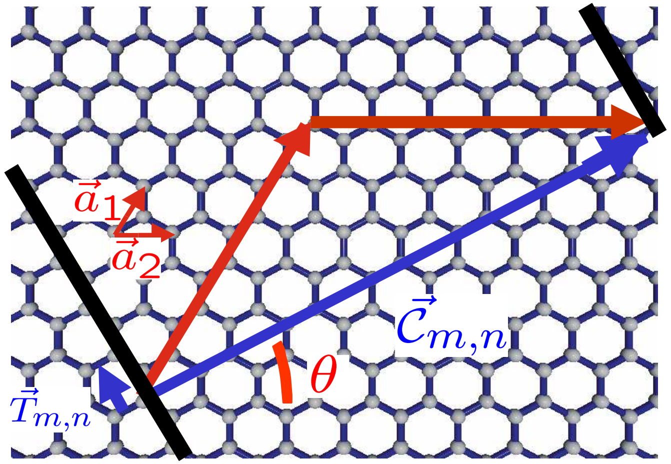


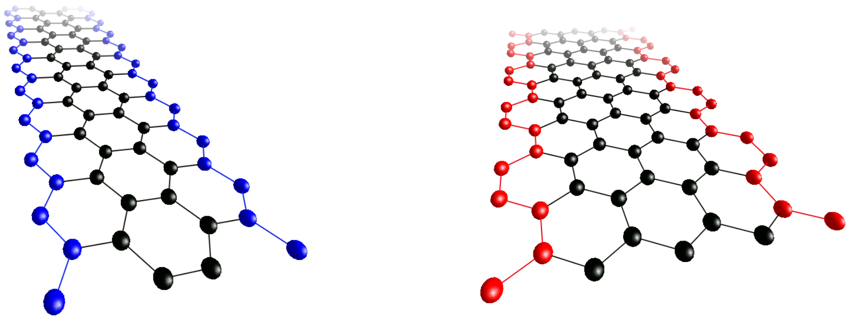




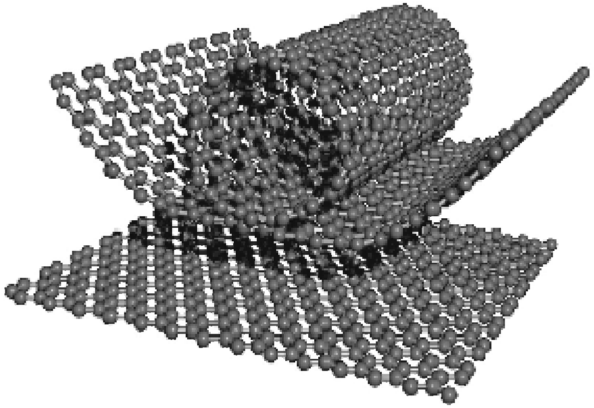
Graphene (ribbons) & Carbon Nanotubes

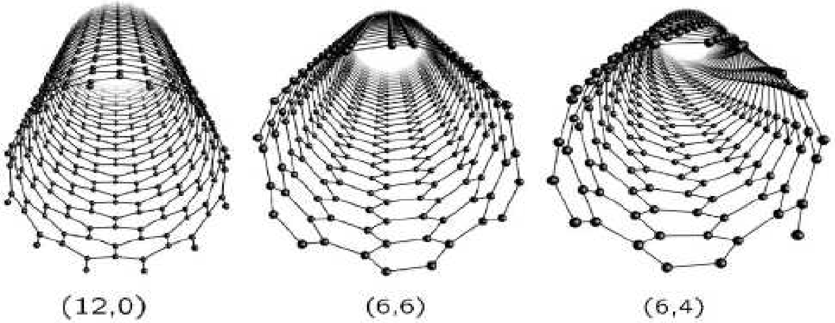




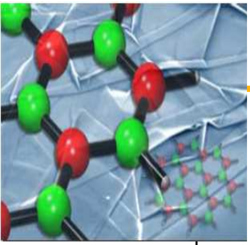


$$\vec{\mathcal{C}}_{m,n} \qquad d_t = \frac{|\vec{\mathcal{C}}_{m,n}|}{\pi}$$





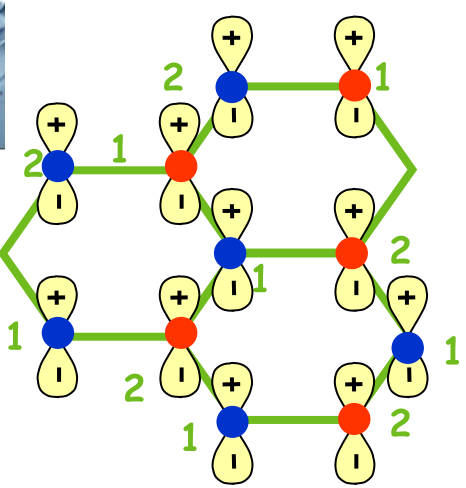
π Effective Model



Hybrid Molecular Orbitales

Cohesion $s, p_x, p_y \equiv \sigma$

Electronic Properties in the vicinity of E_F $p_z \equiv \pi$



2 atoms/ cell γ_0 nearest neighbor orbital overlap p_z

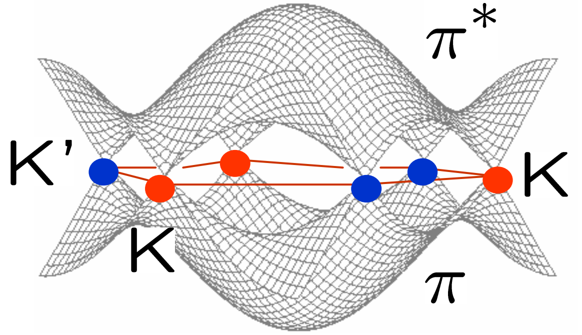
$$H(\vec{k}) = \begin{bmatrix} 0 & f(\vec{k}) \\ f^*(\vec{k}) & 0 \end{bmatrix} \qquad f(\vec{k}) = \gamma_0 \sum_\alpha e^{i\vec{k}.\vec{r}_\alpha}$$

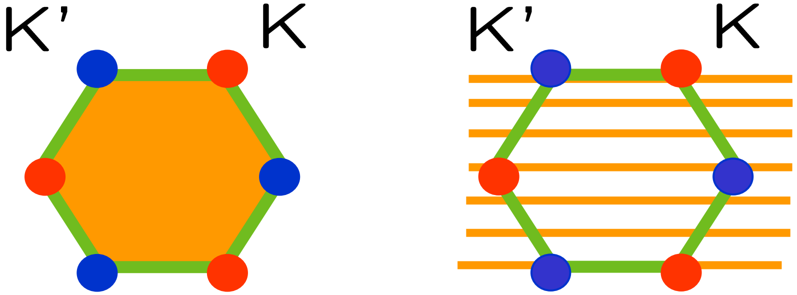
$$E_\pm(k_x,k_y) = \pm\gamma_0\Big(3+4\cos(\frac{\sqrt{3}k_x a}{2})\cos(\frac{k_y a}{2})+2\cos(k_y a)\Big)^{1/2}$$

$E_+(k_x,k_y)$

$E_F = 0$

$E_-(k_x,k_y)$



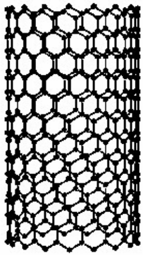


2D

Quasi-1D

1 Brillouin Zone

Nanotubes: Electronic Properties

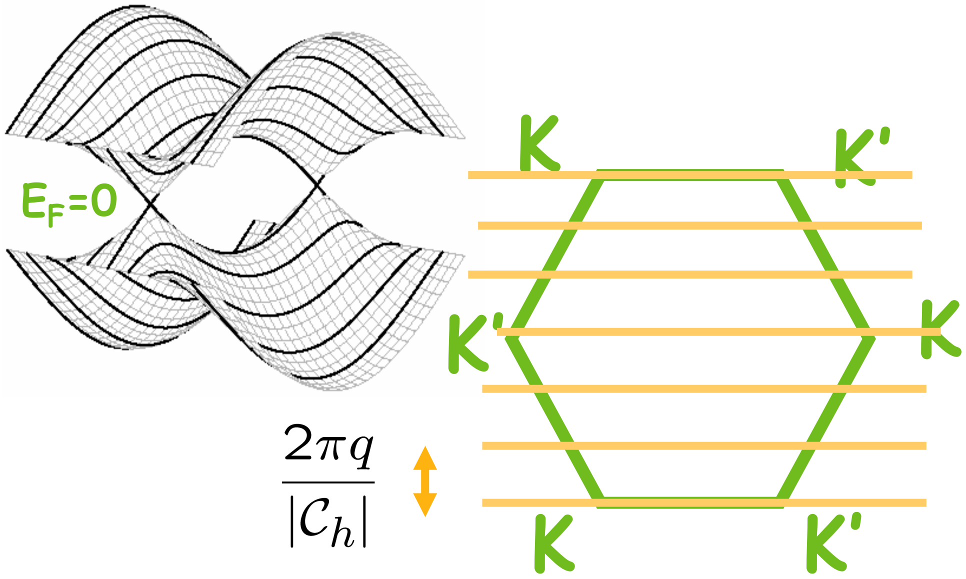


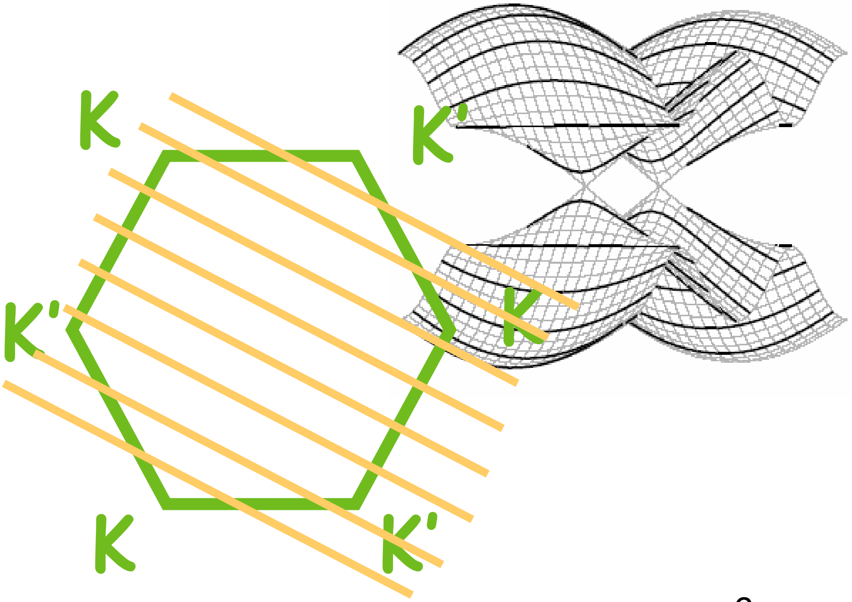
Periodic Boundary conditions

$$-\frac{\pi}{|\vec{T}_{(n,m)}|} \leq k_y(=k) \leq +\frac{\pi}{|\vec{T}_{(n,m)}|} \qquad k_x = \frac{2\pi q}{|\vec{\mathcal{C}}_{(n,m)}|}(q=1,N)$$

Symmetry choice

$$\vec{\mathcal{C}}_{n,n} = n(\vec{a}_1 + \vec{a}_2) \qquad \vec{\mathcal{C}}_{n,m} = (3p \pm 1)\vec{a}_1$$

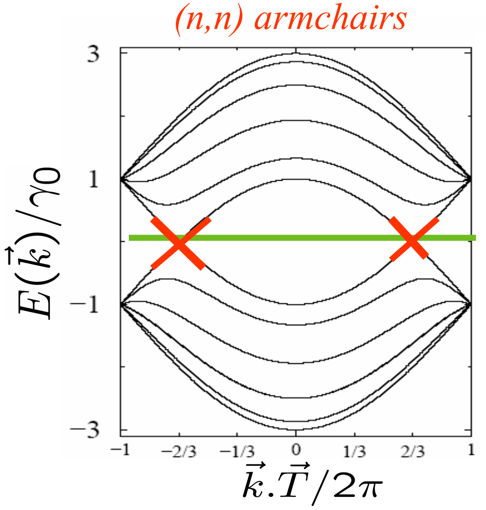


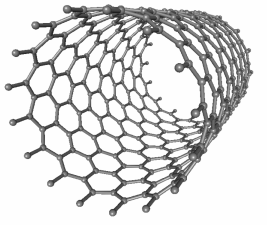


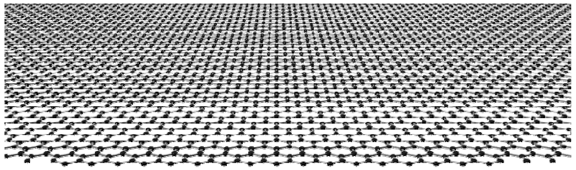
Remark :Massless Dirac Fermions in 2D vs 1D

Nanotubes

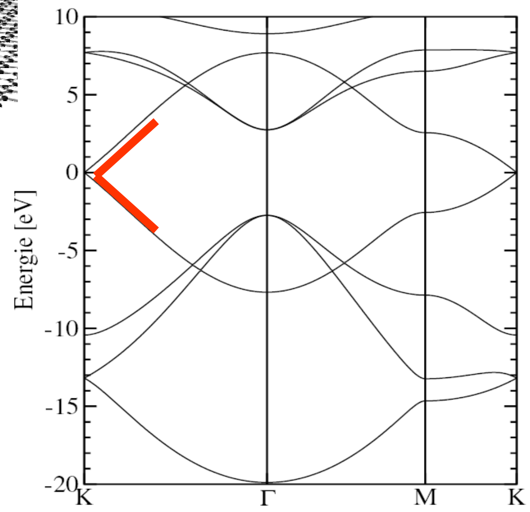
(n,n) armchairs







2D Graphene



Linearization close to Fermi level

$$\vec{Q} = \vec{K}_\pm + \vec{p}/\hbar$$

$$\mathcal{H}_{K_+}(\vec{p}) = v_F \vec{\sigma}.\vec{p}$$

(in the sublattice basis)

Dispersion relation

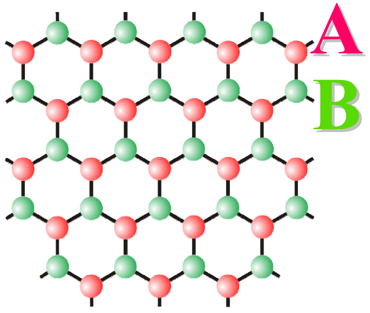
$$E(\vec{p}) = s v_F |\vec{p}| \qquad s = \pm 1$$

$$E(\vec{p}) = s\sqrt{v_F^2 p^2 + m^{*2} c^4}$$

Massless particles

Eigenstates (« pseudospin »)

$$|\Psi_{\vec{p}}^{K_+}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_p^{K_+}(A) \\ \psi_p^{K_+}(B) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta(p)/2} \\ -e^{+i\theta(p)/2} \end{pmatrix}$$



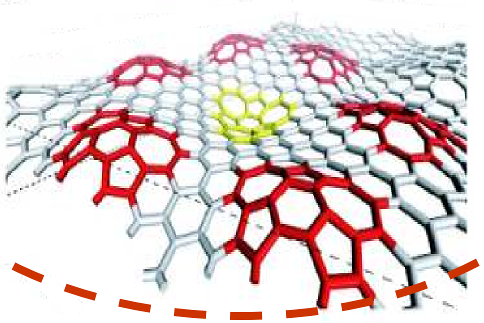
$$|\Downarrow\rangle \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

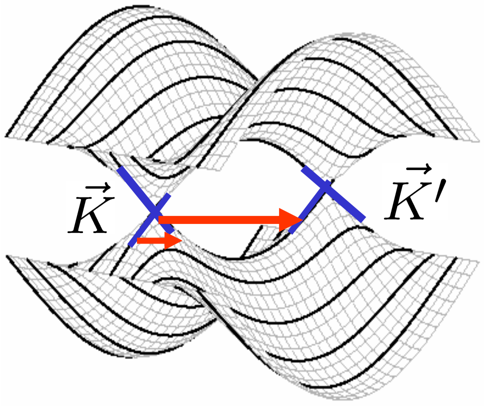
$$|\Uparrow\rangle \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Pseudospin symmetry & backscattering effects

Long range potential

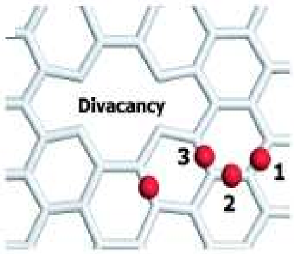
Intravalley scattering





Short range potential

Intervalley scattering



- 1D metallic nanotubes : ***Absence of backscattering,***
- 2D graphene : ***Anti-localization phenomenon,***

Sign inversion of quantum correction ~ S.O coupling effects

1d

=0

$$|\langle\psi_{\mathbf{k},s}|\mathcal{T}|\psi_{\mathbf{k}',s'}\rangle|^2$$

$$\theta_k+\theta_{-k}=\pm\pi$$

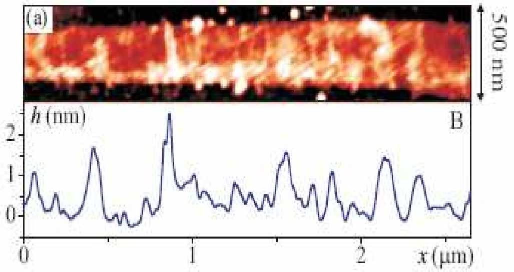
$$\langle s|\mathcal{R}[\theta_k]R^{-1}[\theta_{-k}]|s\rangle = \cos(\theta_k+\theta_{-k})/2,$$

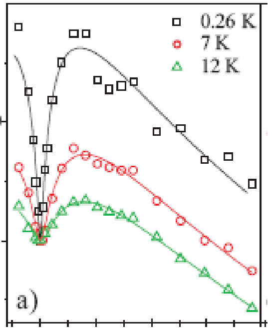
T. Ando, T. Nakanishi and R. Saito,
J. Phys. Soc. Jpn 67, 2857 (1998)

2d

Transition Weak localization

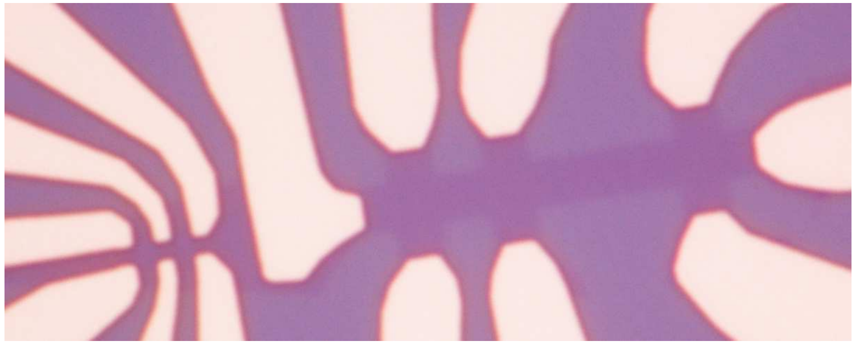
/ Anti-weak localization





E. McCann *et al.*, Phys. Rev. Lett. 97, 146805 (2006)
F.V. Tikhonenko et al., Phys. Rev. Lett. 100, 056802 (2008)

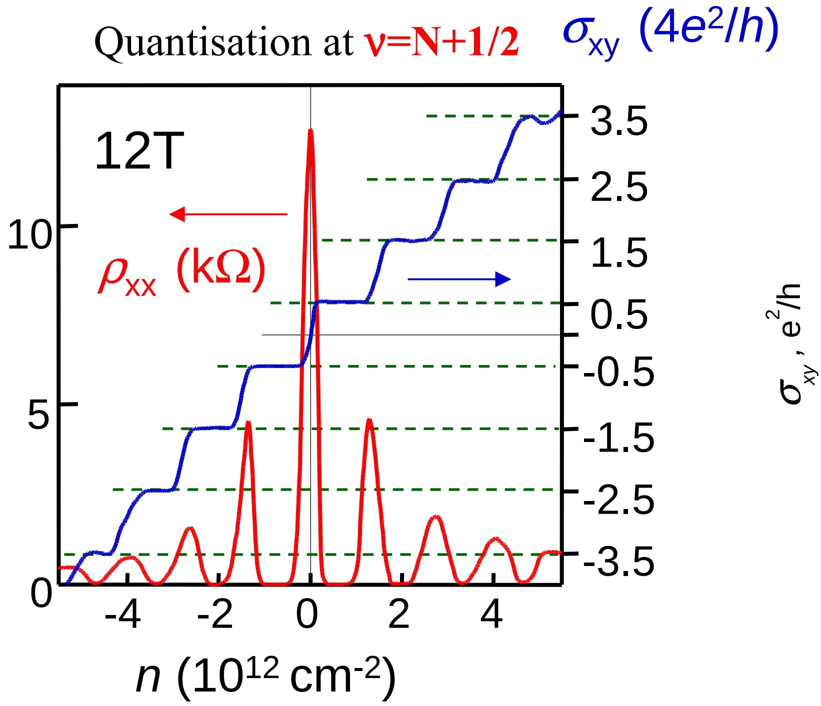
Unconventional Quantum Hall effect



Huge Mobility: 20.000-100.000 cm^2/V·s

(order of magnitude better than silicon)

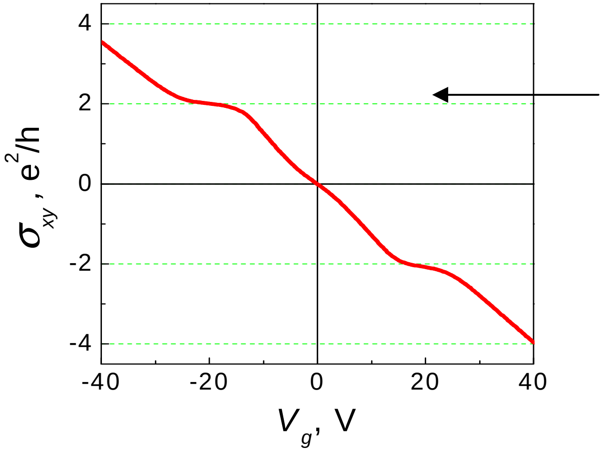
Quantisation at **ν=N+1/2**



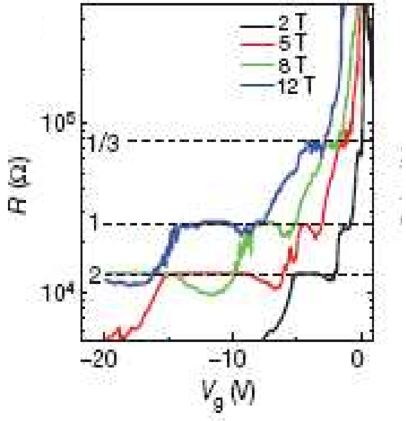
Room Temperature and **low magnetic field**

Integer Quantum Hall effect !

- ➢ Large inter-Landau level distance



$$\sigma_{xy} = 2e^2 / h$$



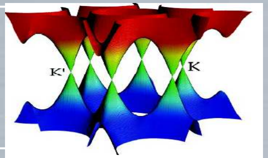
X. Du et al. Nature 2009

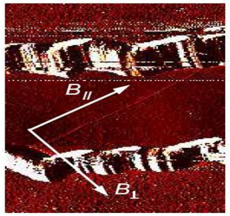
OUTLINE of the TALK

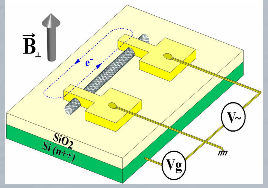
1.

2. **Aharonov-Bohm Effect in CNTs**
   Theory and original controversies
   Magnetic field induced metal-semiconductor transition

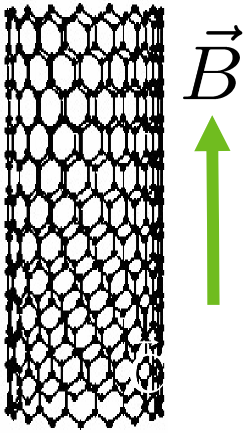
3.







Aharonov-Bohm effects on the Electronic Spectrum



$\{\vec{\mathcal{C}}_h/|\vec{\mathcal{C}}_h|, \vec{T}/|\vec{T}|\}$

Landau gauge

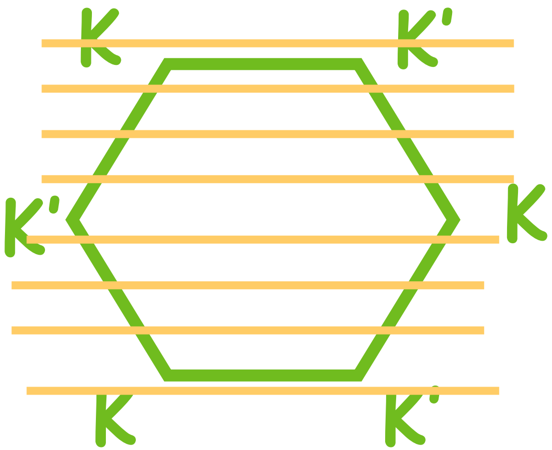
$\vec{\mathcal{A}} = (\phi/|\mathcal{C}_h|, 0)$

Wavefunction

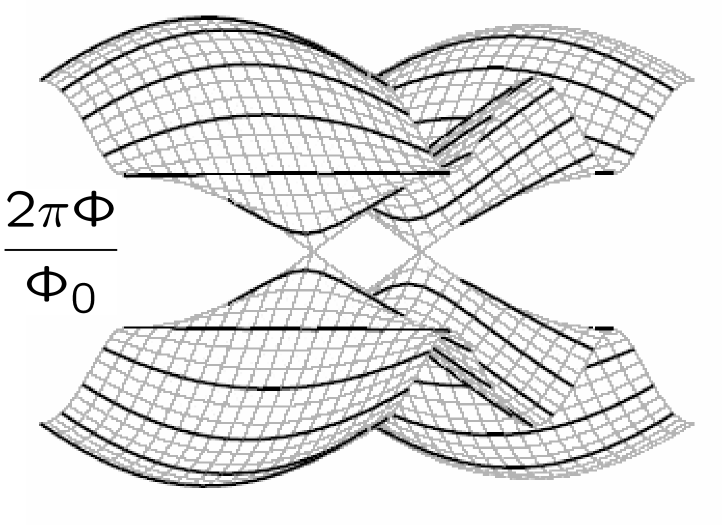
$\Psi \sim e^{ik_y y} e^{i(k_x x + \frac{e}{h}\int \vec{\mathcal{A}}.d\vec{r})}$

$$\Delta\varphi_{\vec{r},\vec{r'}} = \int_0^1 (\vec{r'} - \vec{r}) \cdot (\vec{\mathcal{A}}(\vec{r} + \lambda[\vec{r'} - \vec{r}]))d\lambda$$
$$= i(x - x')\phi/|\vec{\mathcal{C}}_h|$$

$$\boxed{\delta\vec{k}(\phi)\cdot\vec{\kappa}_x = \delta\vec{k}(0)\cdot\vec{\kappa}_x + 2\pi\phi/(\phi_0|\vec{\mathcal{C}}_h|)}$$



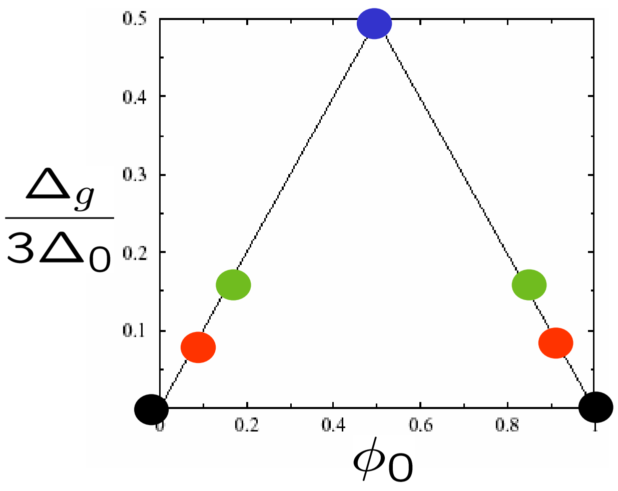
$$k_x = \frac{2\pi q}{|\mathcal{C}_h|} + \frac{2\pi\Phi}{\Phi_0}$$

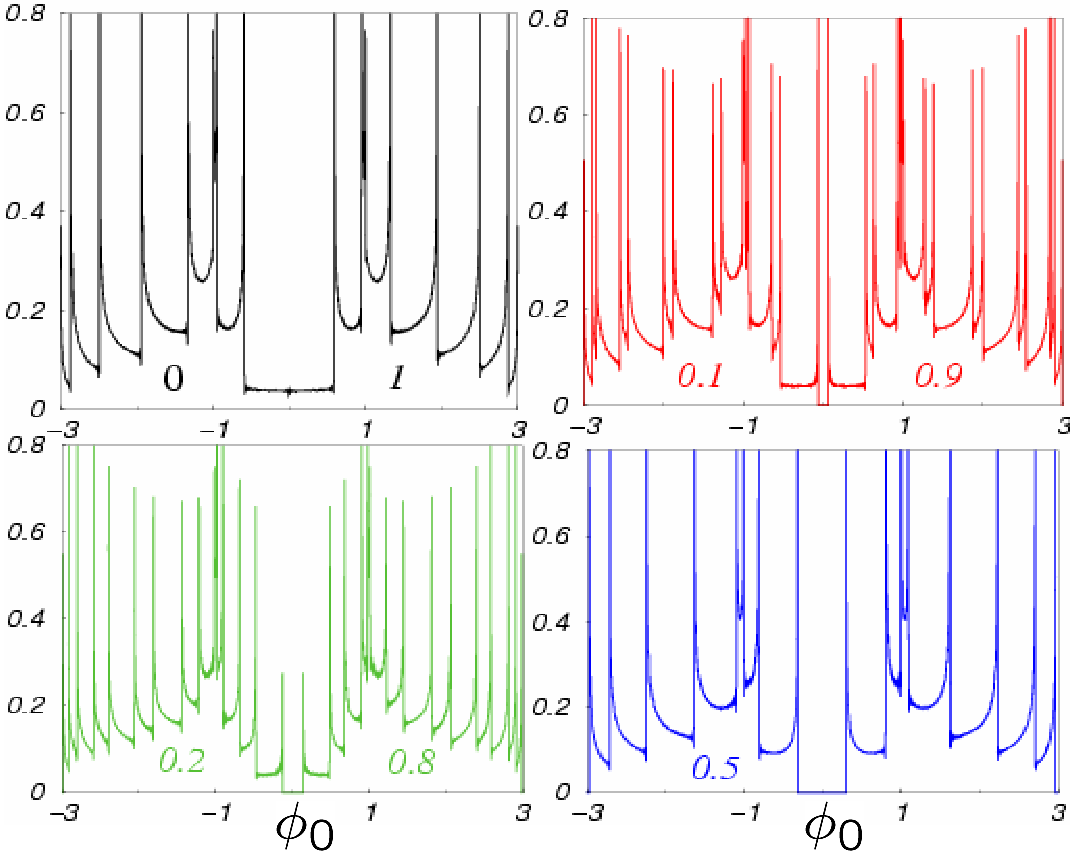


Magnetic field driven spectral changes

ϕ_0 Periodic Gap-oscillations   Van-Hove Singularities modulations

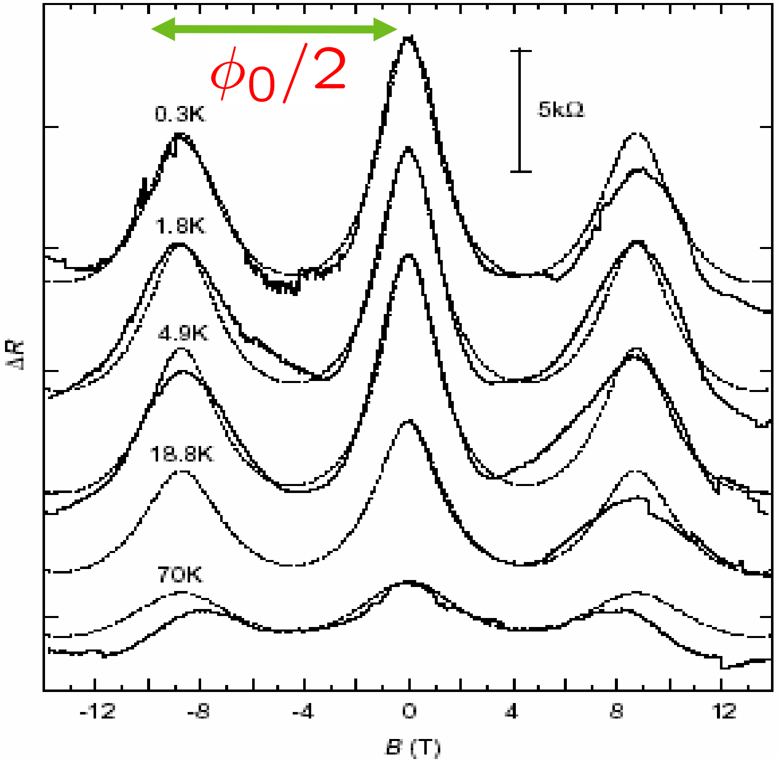
$$\Delta_g = \begin{cases} 3\Delta_0 \dfrac{\phi}{\phi_0} & \text{if} \quad 0 \leq \phi \leq \dfrac{\phi_0}{2} \\ 3\Delta_0 \left| 1 - \dfrac{\phi}{\phi_0} \right| & \text{if} \quad \dfrac{\phi_0}{2} \leq \phi \leq \phi_0 \end{cases}$$

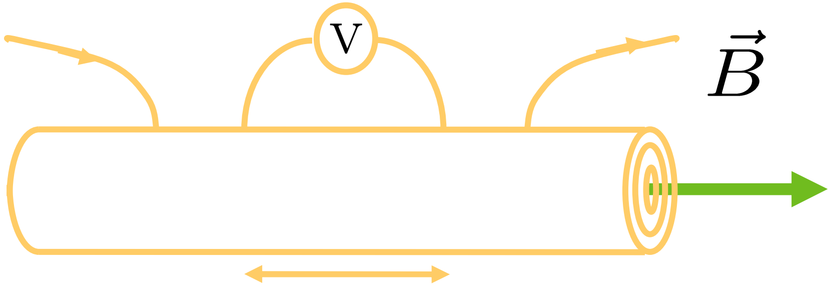




H. Akiji and T. Ando, J. Phys. Soc. Jpn 62, 2470 (1993)
H. Akiji and T. Ando, J. Phys. Soc. Jpn 65, 505 (1996)
S.R., G. Dresselhaus, M. Dresselhaus, R. Saito, PRB 62, 16092 (2000)

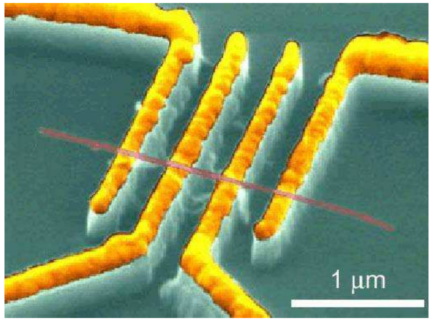
Magnetotransport in nanotubes : first experiments





A. Bachtold et al, **Nature 397, 673 (1999)**

* Negative Magnetoresistance
* $\phi_0/2$ -periodic oscillations



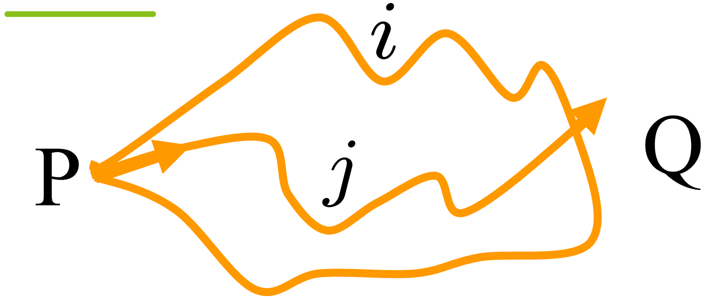
Weak localization (AAS oscillations)

Diffusive regime (small elastic mean free path)

Weak localization phenomenon

Beyond the diffusive regime

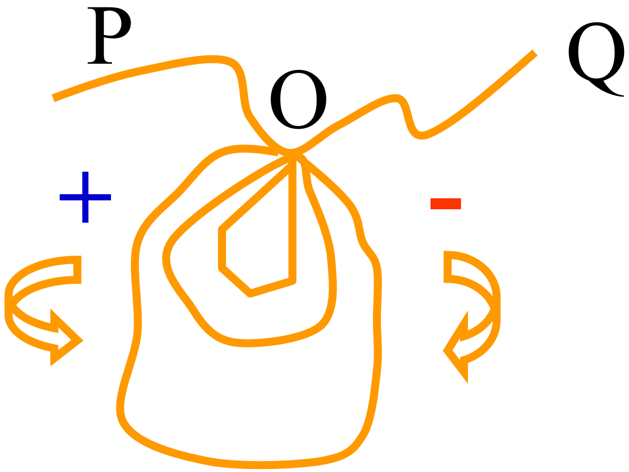
Quantum conductance $G = \frac{2e^2}{h}\mathcal{P}_{P\to Q}$



$$\mathcal{P}_{P\to Q} = \sum_i |\mathcal{A}_i|^2 + \sum_{i\neq j} \mathcal{A}_i\mathcal{A}_j e^{i(\alpha_i-\alpha_j)}$$

Classical term

Interference term



Time-reversed trajectories interfere constructively

$$\mathcal{P}_{O\to O} = \left|\mathcal{A}_+ e^{i\alpha_+} + \mathcal{A}_- e^{+i\alpha_-}\right|^2 = 4\left|\mathcal{A}_0\right|^2$$

Quantum interference effects

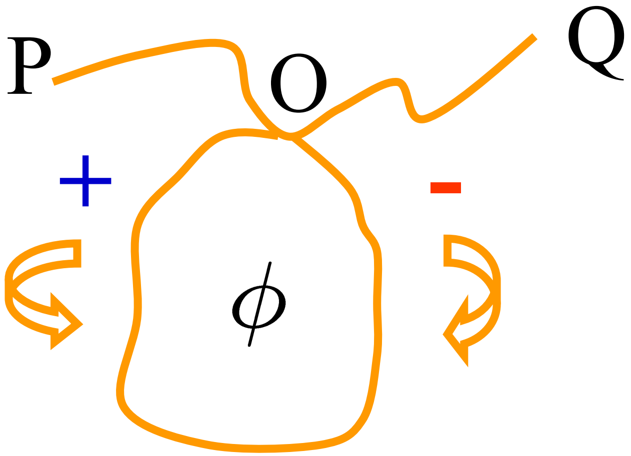
↓

Enhanced return probability to the origin

Increase of quantum resistance

WL & Aronov-Altshuler-Spivak oscillations

Switching on Perpendicular B



$$\alpha_\pm = \pm \frac{e}{\hbar} \oint \vec{A}.d\vec{r}$$

$$\mathcal{P}_{O\to O} = 4\left|\mathcal{A}_0\right|^2$$

$$\Downarrow$$

$$\left|\mathcal{A}_0\right|^2 \left|1 + e^{i(\alpha_+ - \alpha_-)}\right|^2 = 2\left|\mathcal{A}_0\right|^2 \left(1 + \cos\frac{2\pi\phi}{\phi_0/2}\right)$$

NEGATIVE MAGNETORESISTANCE

$\phi_0/2$ -periodic oscillations

Theory: Altshuler,Aronov & Spivak **JETP 1981**

Experiment: Sharvin & Sharvin **JETP 1981**

B-dependent diffusion coefficient

Average over the spectrum

$\ell_e \leq L_{\text{tube}}$

Negative MR

$\frac{\Phi_0}{2}$ **-oscillations (type AAS)**

$L_{\text{tube}} \geq \ell_e \geq |\mathcal{C}_h|$

Negative MR

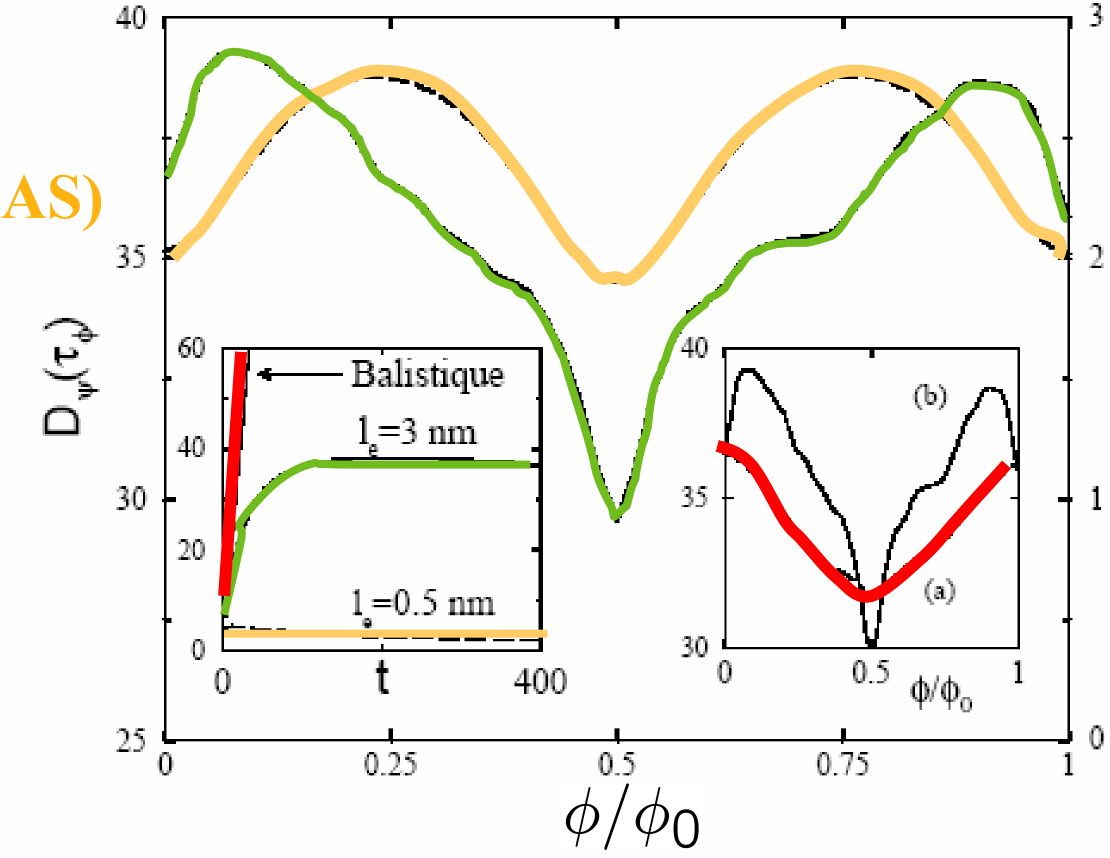
Φ_0 **-oscillations (AB)**

$\ell_e \geq L_{\text{tube}}$

Positive MR

Φ_0 **-oscillations (AB)**

Metallic tube
(Anderson-type disorder)



SR, F. Triozon, A. Rubio, D. Mayou,

PRB 64, 121401 (2001)

Magnetoconductance for parallel fields

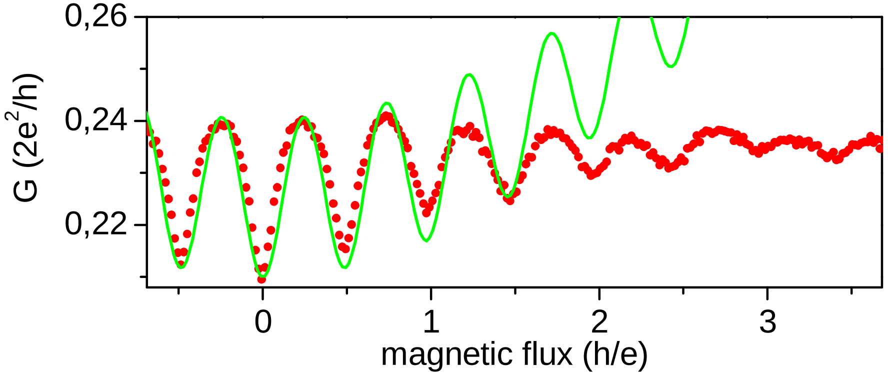
Tube length: 1.4 µm
diameter: ~ 36 nm
$T = 4.5$ K
Périodicity en ϕ_0

$$\Delta B = \frac{\phi_0}{\pi r^2} = 4.1\,\mathrm{Tesla}$$

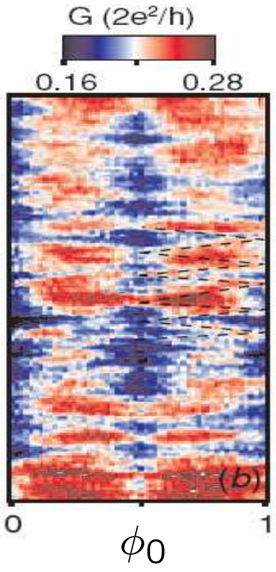
Ch. Strünk
(regensburg, Germany)

Weak localization signatures

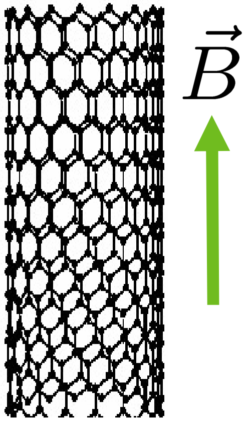
Negative MR
Oscillations AAS $\frac{\phi_0}{2}$



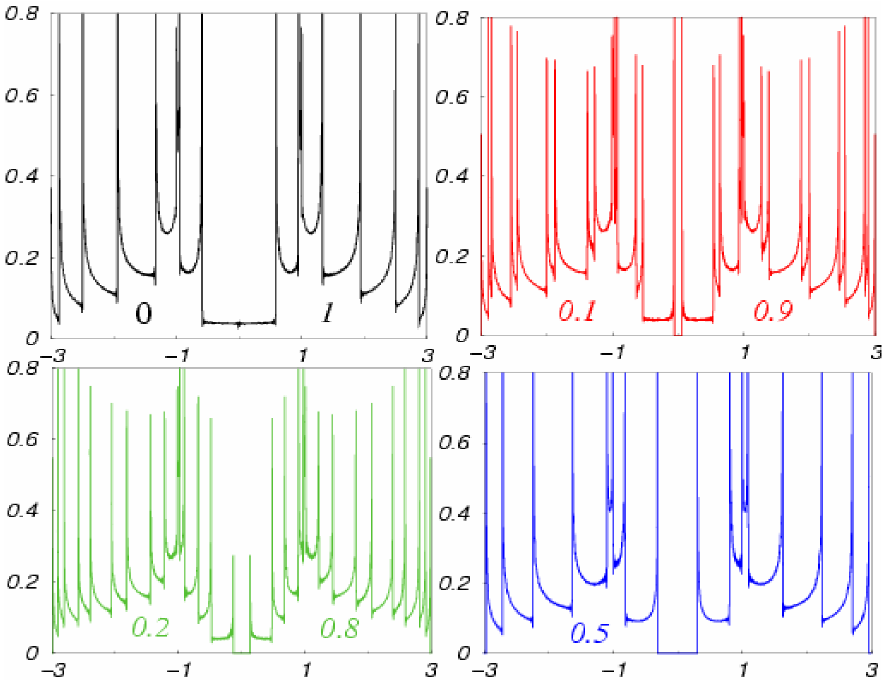
Field dependent additional features ?



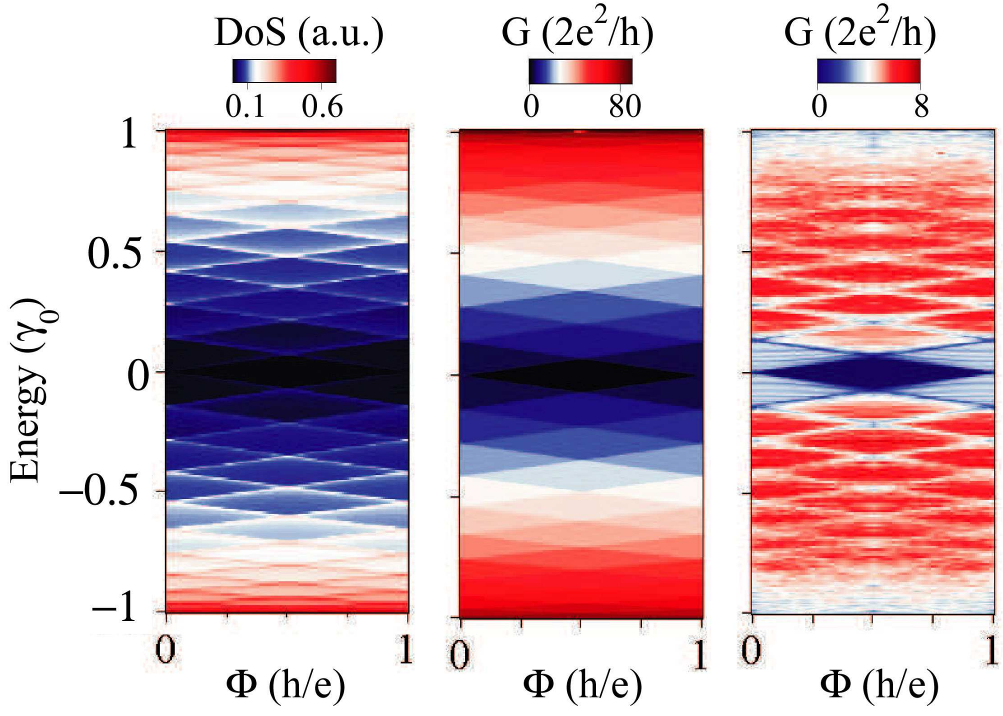
B-dependent bandstructure features



Parallel Magnetic field
Van-hove singularity splitting
Orbital degeneracy breaking



S.R., G. Dresselhaus, M. Dresselhaus, R. Saito, PRB 62, 16092 (2000)



no disorder

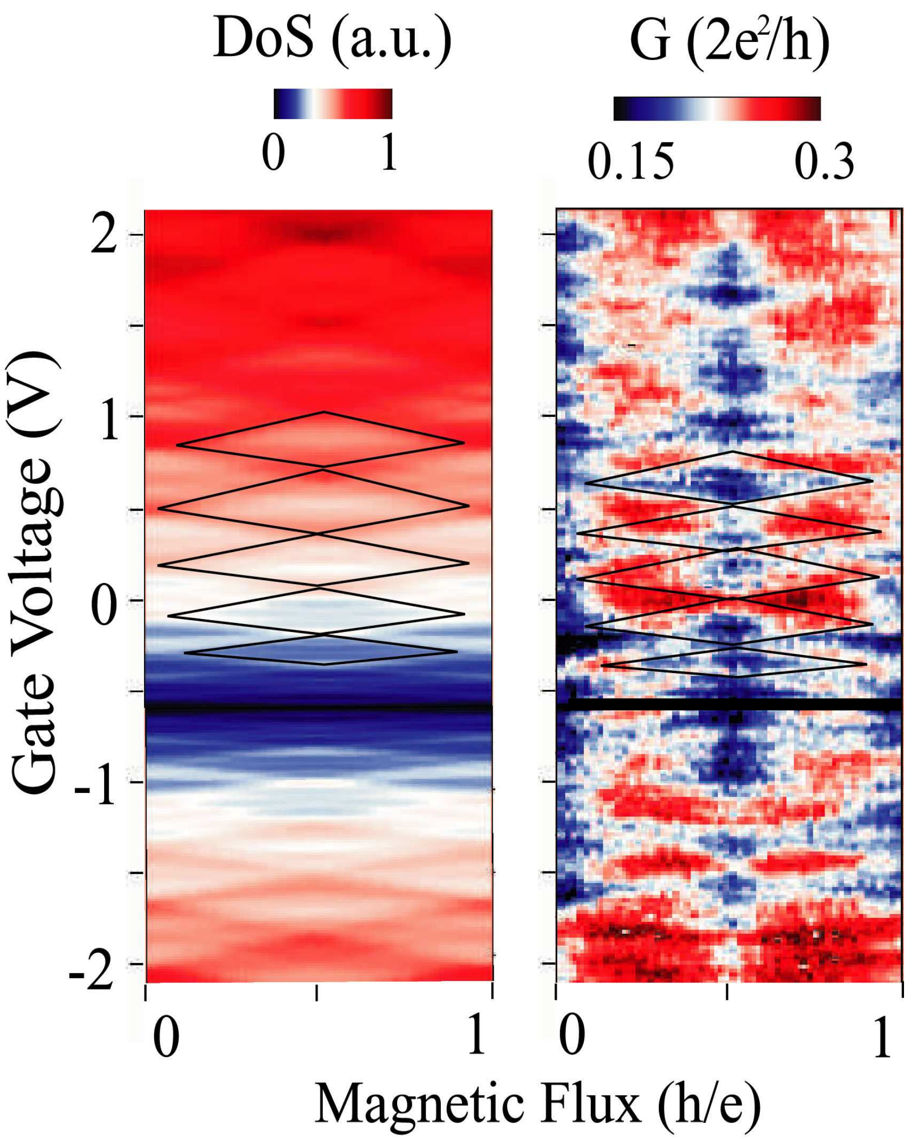
no disorder

with disorder

Field-dependent DoS + conductance for (22-22) 3nm metallic nanotube

Comparison with experiments

Density of states for a (260,260) metallic nanotube (diameter ~ 36 nm)

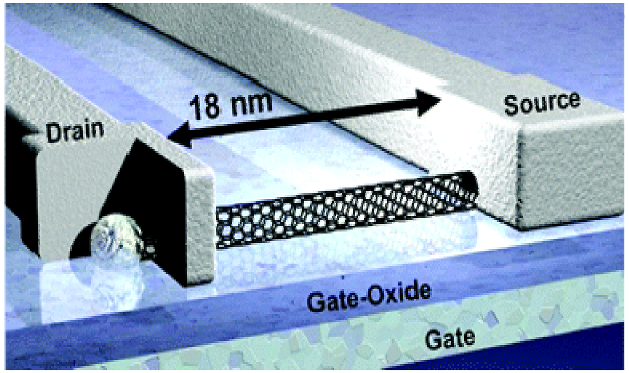


Experimental data
For conductance

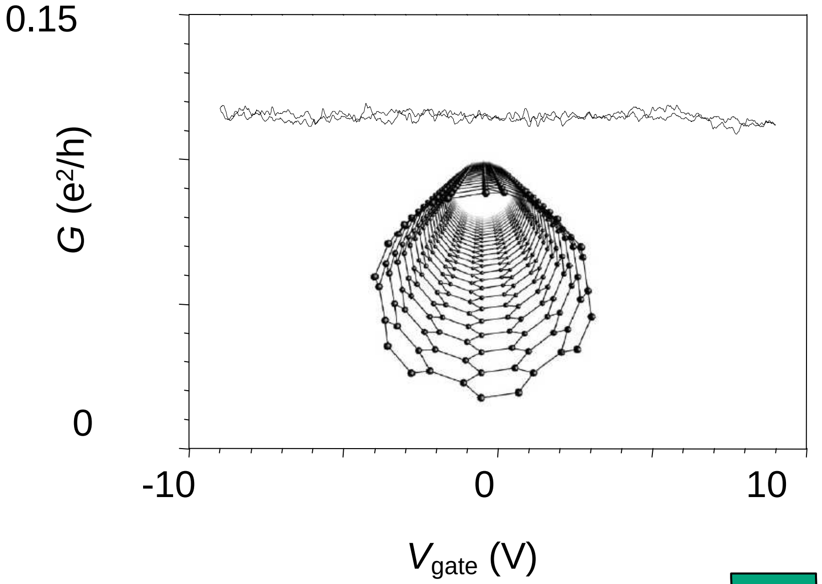
Systems too disordered and larger diameter to see Gap opening…

C Strunk, B Stojetz and SR
Semicond. Sci. Technol. **21** (2006) S38–S45

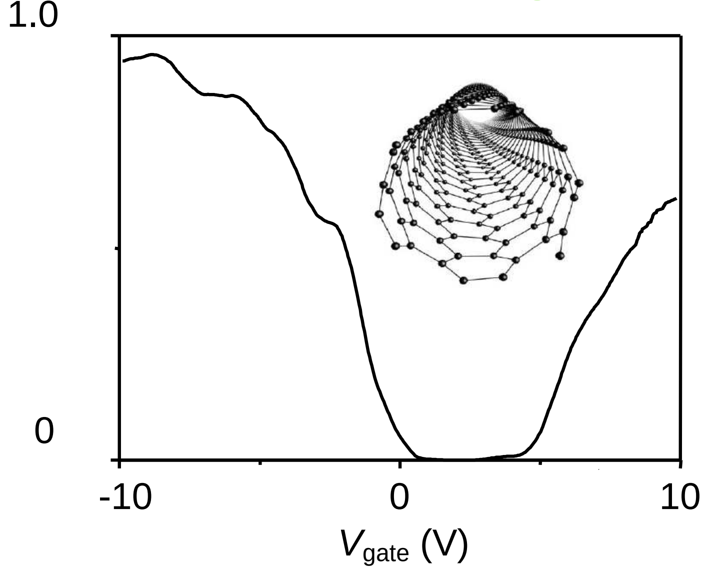
CNT-based device characteristics



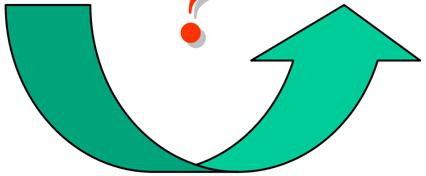
Metallic nanotube



Semiconducting nanotube

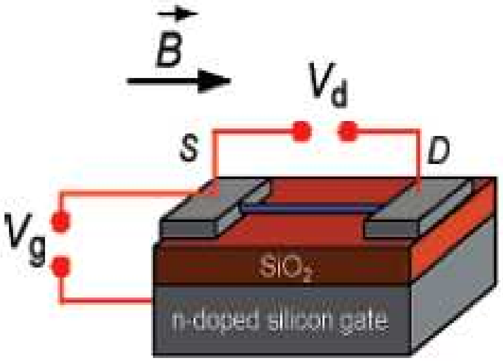


Conducting nanoscale interconnects

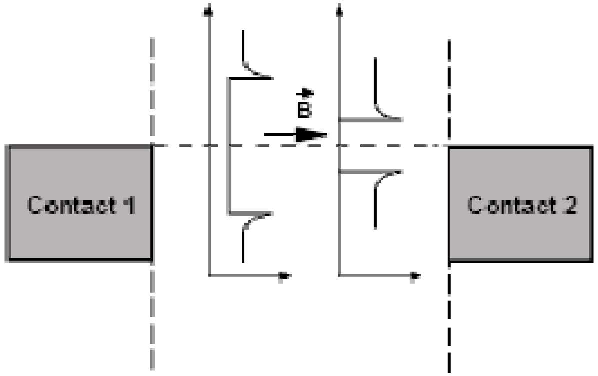


Field Effect Transistors

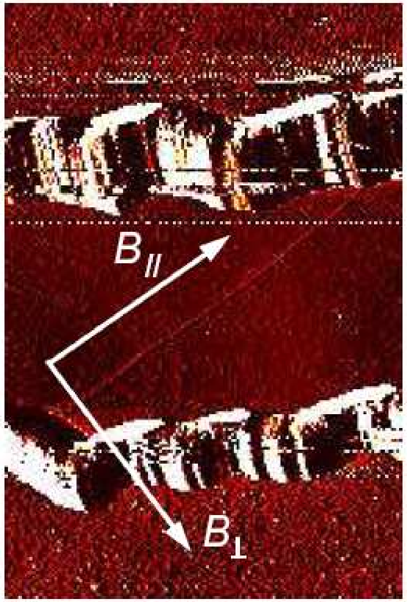
Basic Principle to engineer a B-modulated FET ?

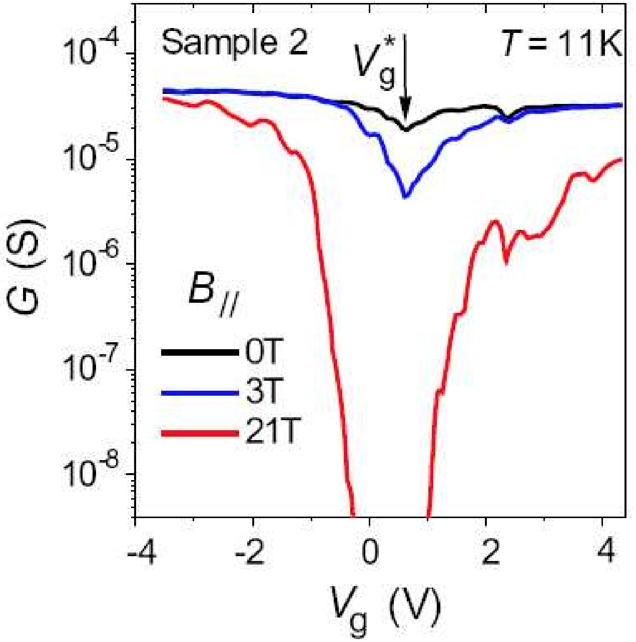


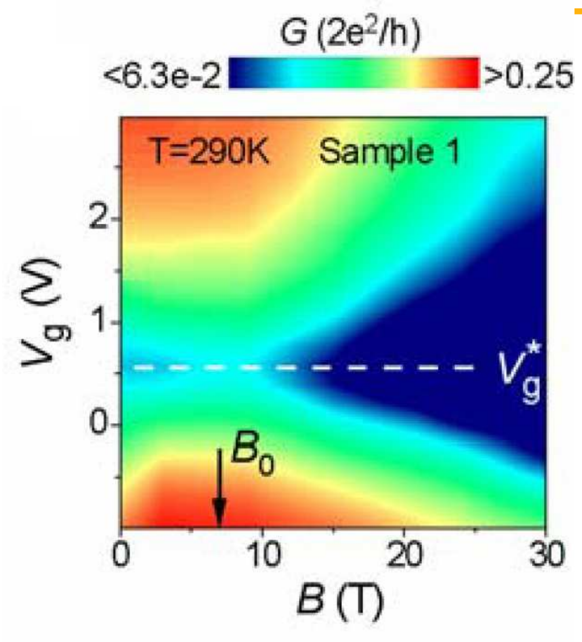
?

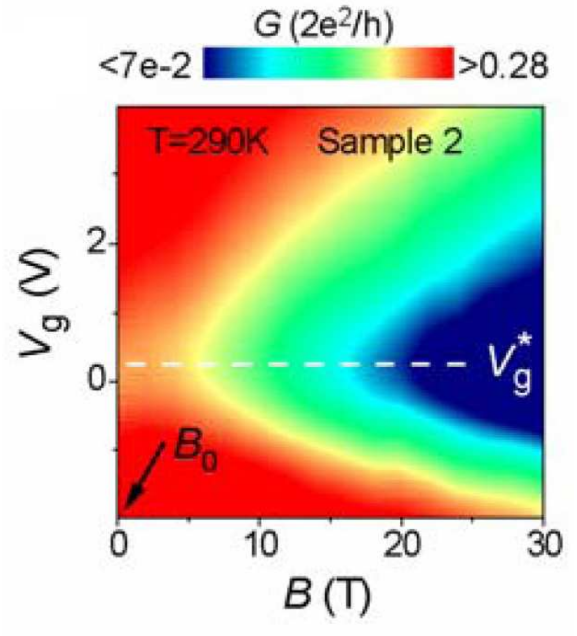


G Fedorov, A Tselev, D Jiménez, S Latil, N Kalugin, P Barbara, D Smirnov, SR,
Nano Lett. 7, 960 (2007)





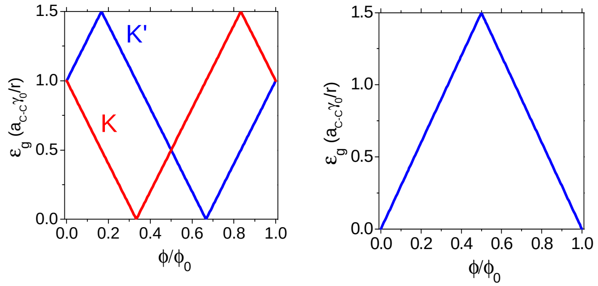




Chirality dependent effects

-)Tight-binding calculations for all possible chiralities (diam ~ 1-2 nm)

Chirality identification

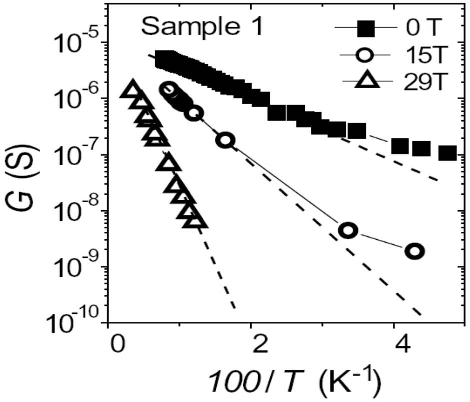


Charge transport mechanism

-) Temperature-dependent G reveals that charge transport is dominated by a

Tunneling regime through a Schottky barrier (B-dependent features)

Arrhenius plots



OUTLINE of the TALK

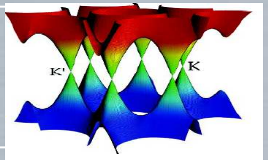
1.

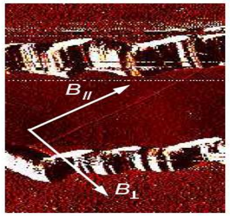
2.

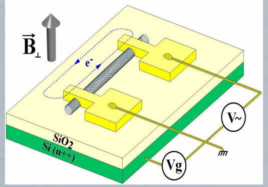
3. **Landau levels in CNTs**

Fabry-Perot regime

Propagative Landau levels and Fermi level pinning







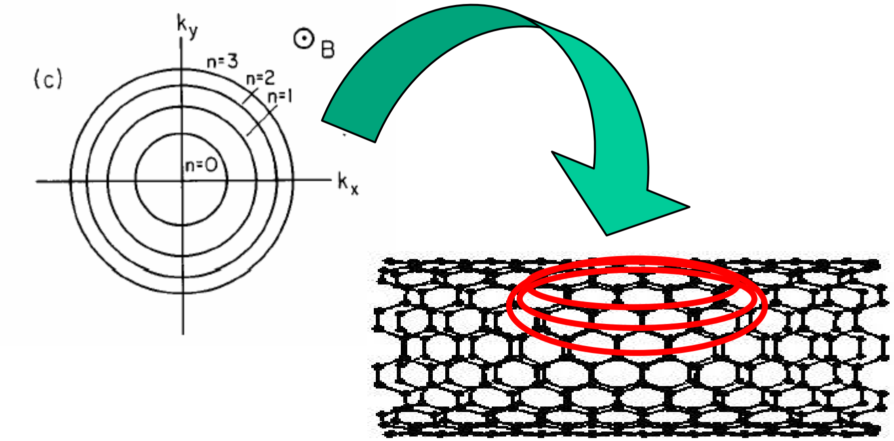
CENTRE D'INVESTIGACIÓ
EN NANOCIÈNCIA
I NANOTECNOLOGIA

CAMPUS UAB. BELLATERRA. BARCELONA

Perpendicular Magnetic field

An experimental challenge

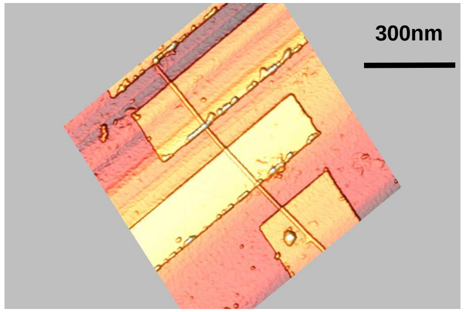
☞ To fit the Landau radius on the surface the CNT



$$\boxed{\nu^2 = \left(\frac{R}{l_B}\right)^2 = \frac{R^2 e}{\hbar} B \gg 1}$$

d (nm)	SWCNT	MWCNT
B(ν=1) (T)	2633	30

☞ To work on clean CNT for a clear observation of Landau quantization

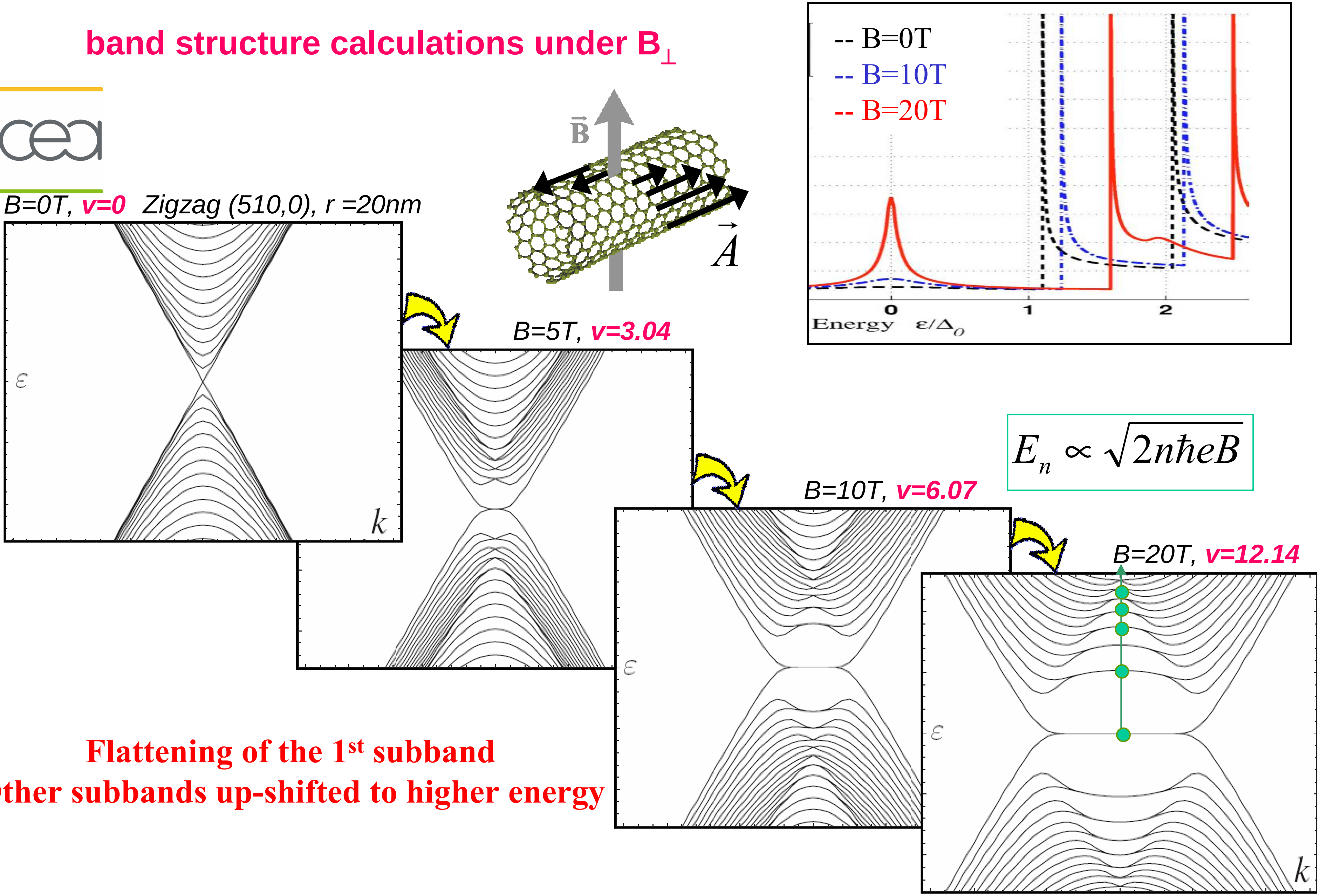


$$\boxed{l_e \gg l_B}$$

In clean MWCNT, $l_e \approx$ few 100nm

Perpendicular Magnetic field

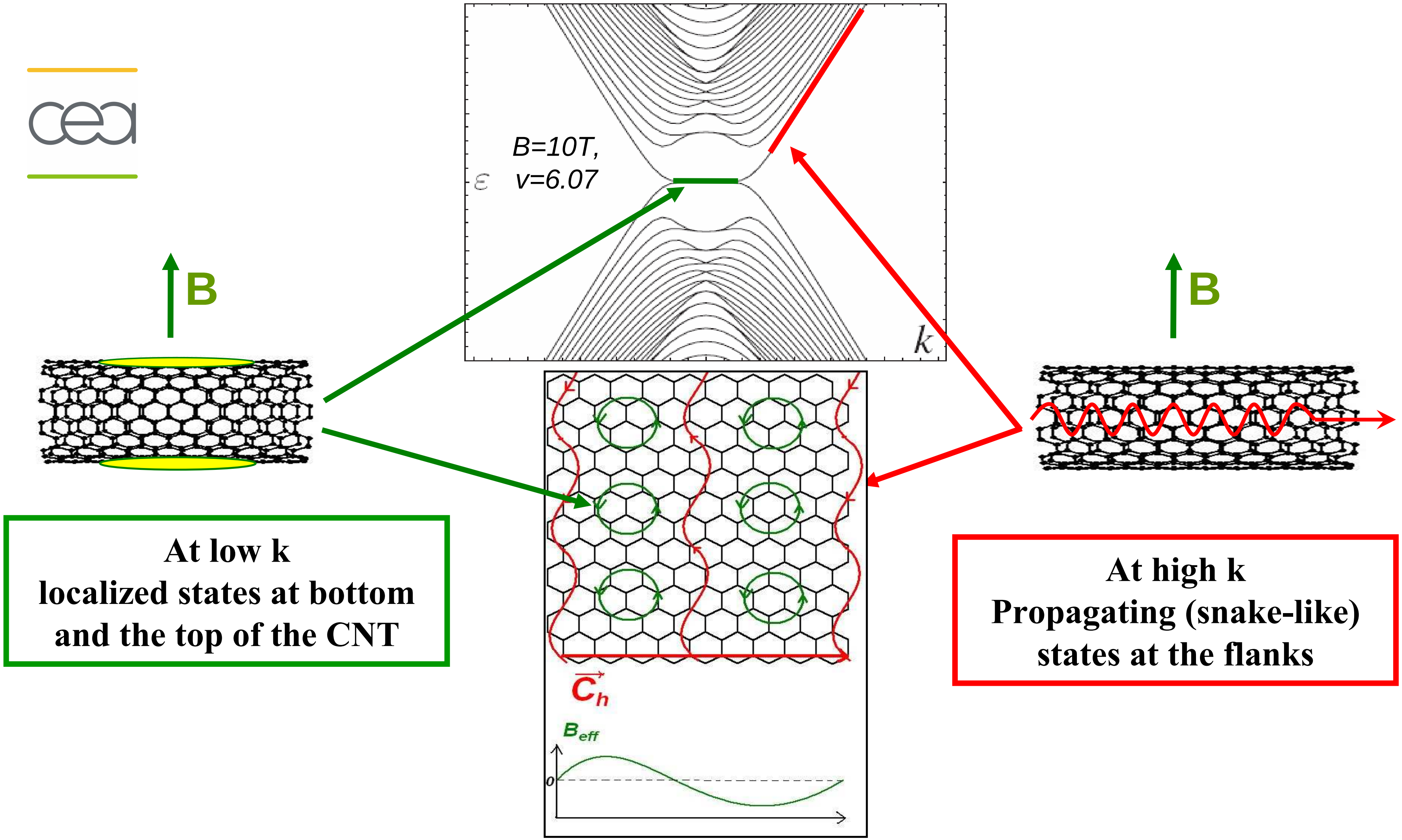
band structure calculations under $B_\perp$



$$E_n \propto \sqrt{2n\hbar eB}$$

Flattening of the 1st subband
Other subbands up-shifted to higher energy

Some pictures of Landau states

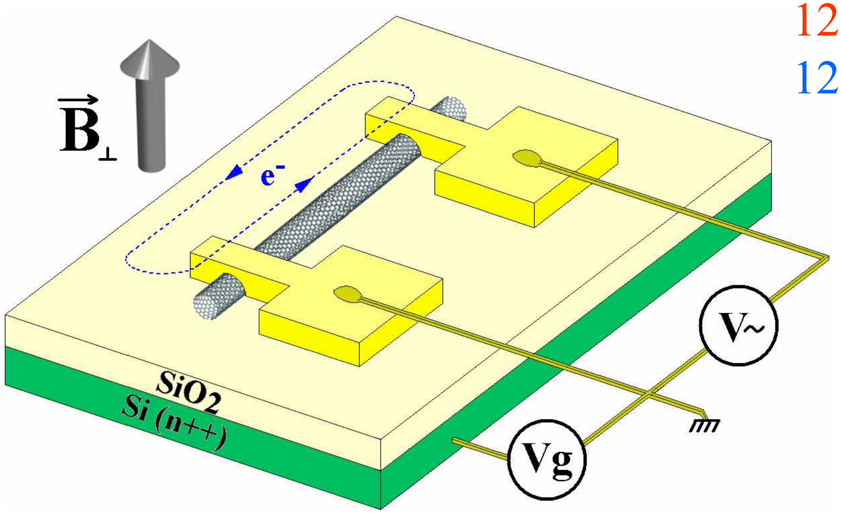


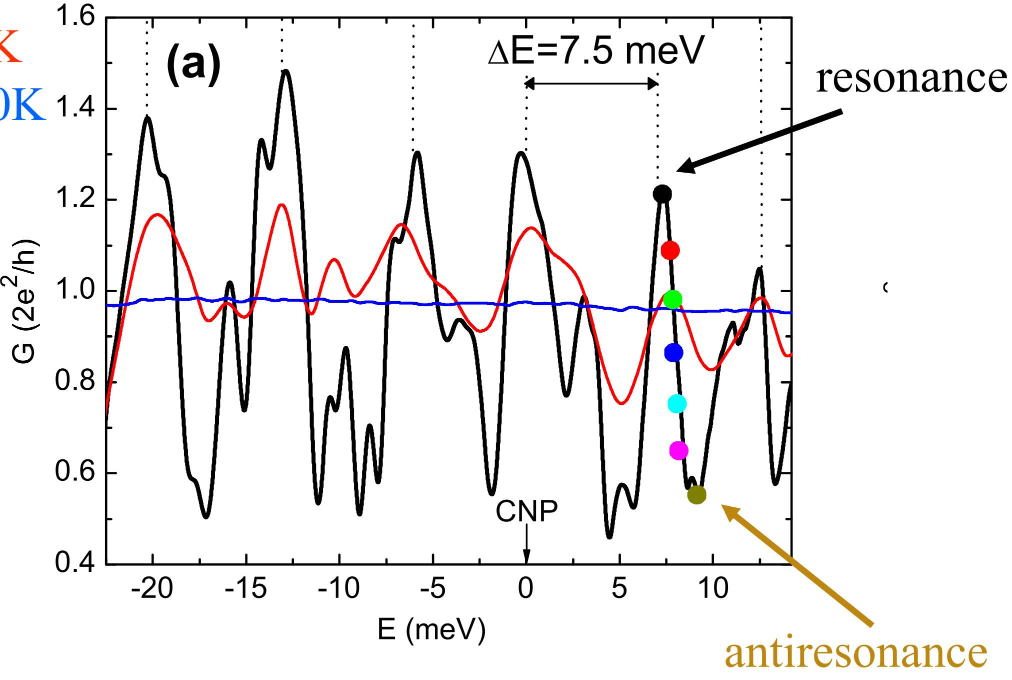
At low k
localized states at bottom
and the top of the CNT

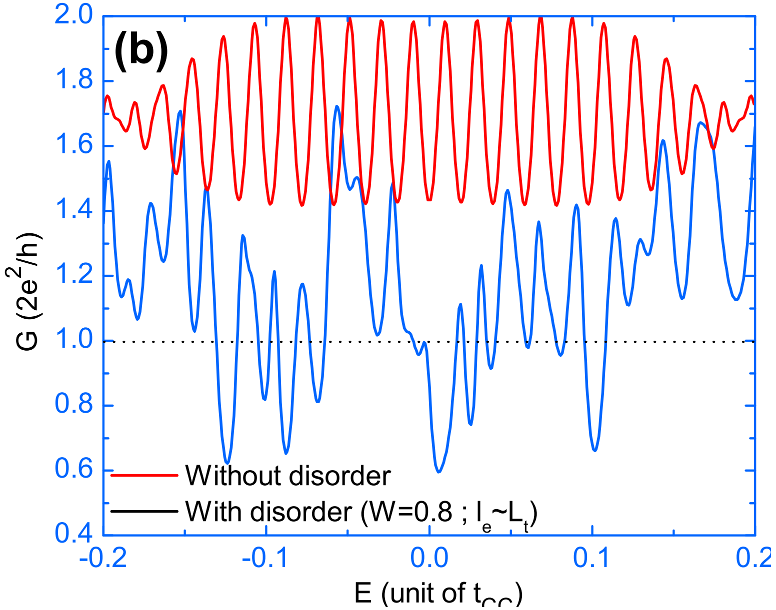
At high k
Propagating (snake-like)
states at the flanks

D2

Ballistic Metallic Tube : Fabry-Pérot Cavity







- **Coherent low T regime.**
- **Fabry-Perot oscillation.**

Simulation (Landauer)

$$\delta E = h v_F / 2L \approx 7.5 meV$$

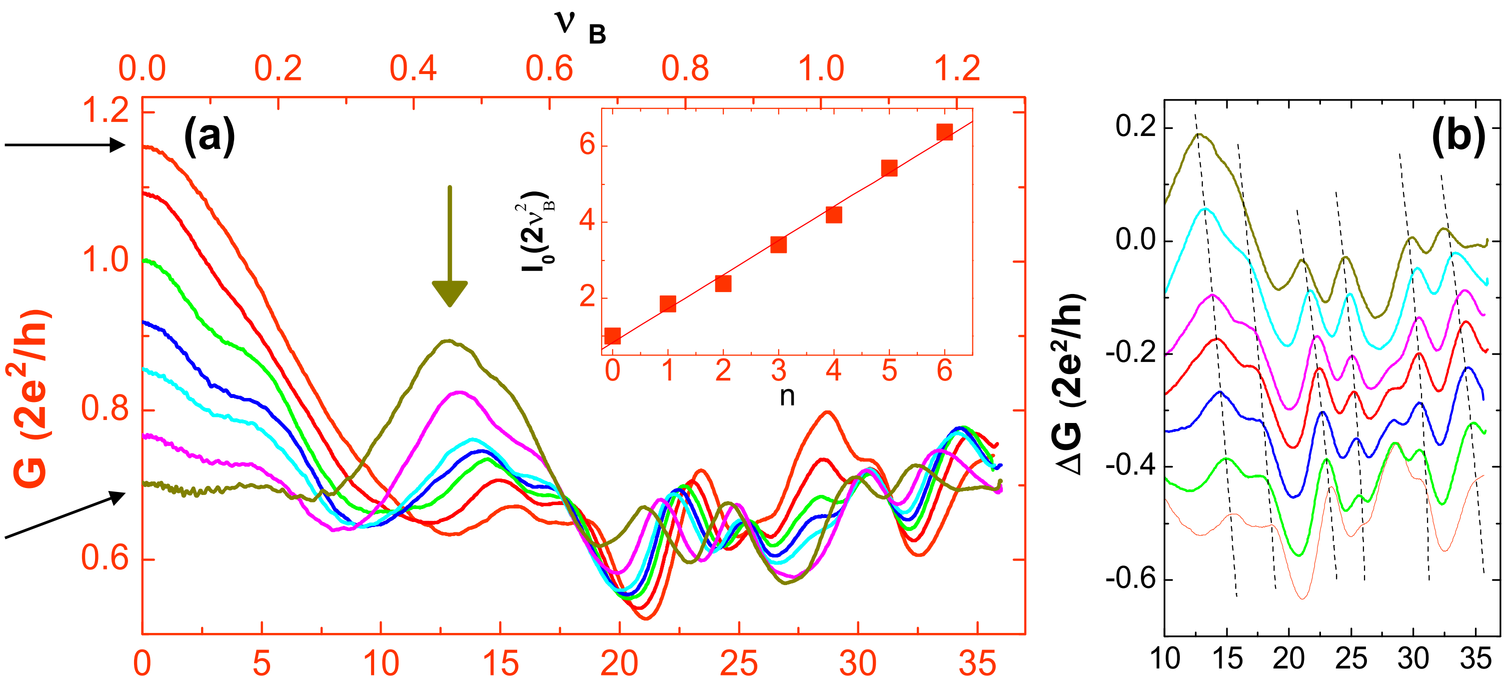
D2 Expérience interférométrique : dépendance de la phase de la fonction d'onde dans une cavité résonante. Oscillations de Fabry-Pérot.
DRFMC; 26/03/2008

D4

Magnetic-field induced Modulation of interferences

Maximum of FP resonance

Minimum of FP resonance



- **Modification of band structure under B_{perp} field :**

$$E_{\pm}(k,\nu_B) = \pm \frac{\hbar v_F}{I_0\left(2\nu_B^2\right)}\left|k - k_F\right|$$

- **Maximas of G each time the matching phase condition is recovered :**

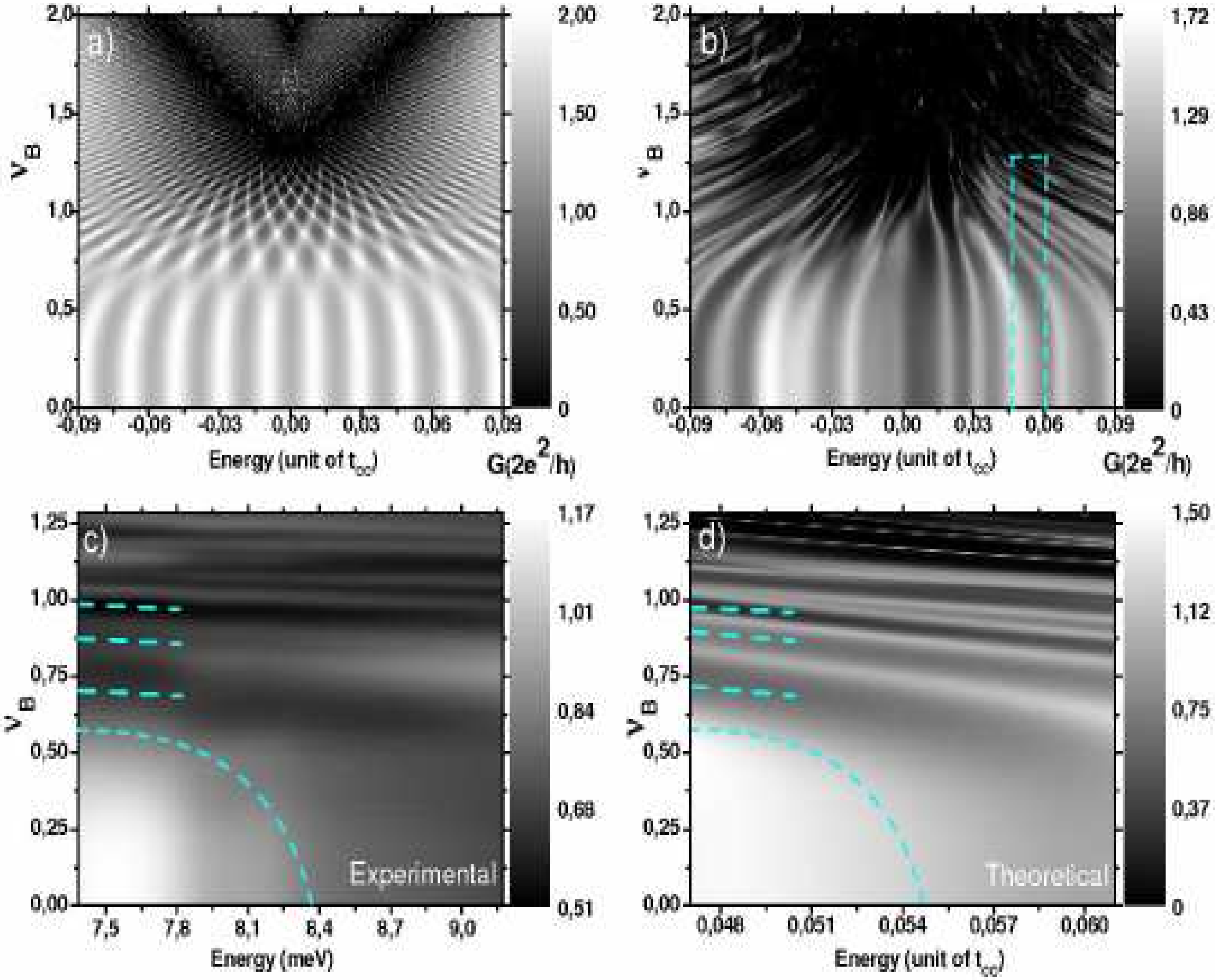
$$\delta k(\nu_B)L = p\pi$$

Diapositive 28

D4 Mécanisme : pente relation dispersion diminue (structure de bande modifiée sous champ) et condition d'accord de phase modifée.
Résultats cohérents qualitativement : fonction de Bessel retrouvée en fonction de l'ordre de la résonance. Pente des courbes à un ordre donné dB/dE bon signe et bon ordre de grandeur.
Oscillation apréiodiques pilotées par la fonction de Bessel.
Vg entrre max et min d'une résonance. Pente bas champ : effet de contact ?
DRFMC; 26/03/2008

Signature of Landau level formation on conductance

With Disorder



B. Raquet, R. Avriller, B. Lassagne, S. Nanot, W. Escoffier, JM Broto, S.R...

Phys. Rev. Lett. 101, 046803 (2008)

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David Jimenez (UAB)

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Benjamin Lassagne, Sebastien Nanot, Bertrand Raquet

CNRS INSA UNIVERSITE PAUL SABATIER

LNCMI

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ICN Institut Català de Nanotecnologia

CSIC

cin2 CENTRO DE INVESTIGACION EN NANOTECNOLOGIA

Alexander von Humboldt
Stiftung/Foundation