Threshold of a Random Laser with Cold Atoms

Luis S. Froufe-Pérez [(1)], W. Guerin [(2)], R. Carminati [(3)], R. kaiser [(4)].

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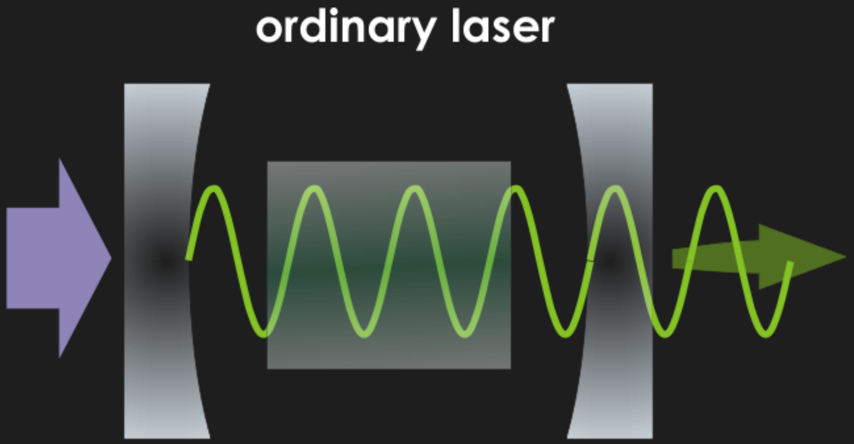
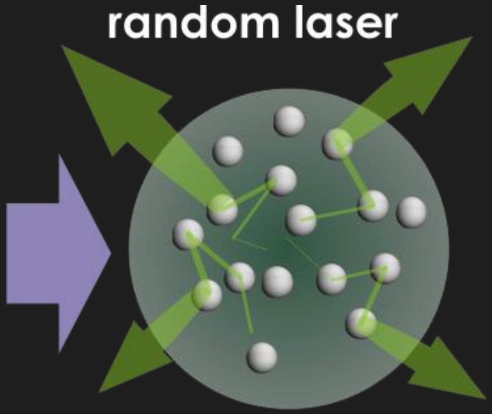
(2) Institute non linéaire de Nice, CNRS-U- Nice. France.
(3) Institut Langevin, ESPCI ParisTech, CNRS. France.

Random Lasing



Modes + Gain ➔ LASER

Random Lasing



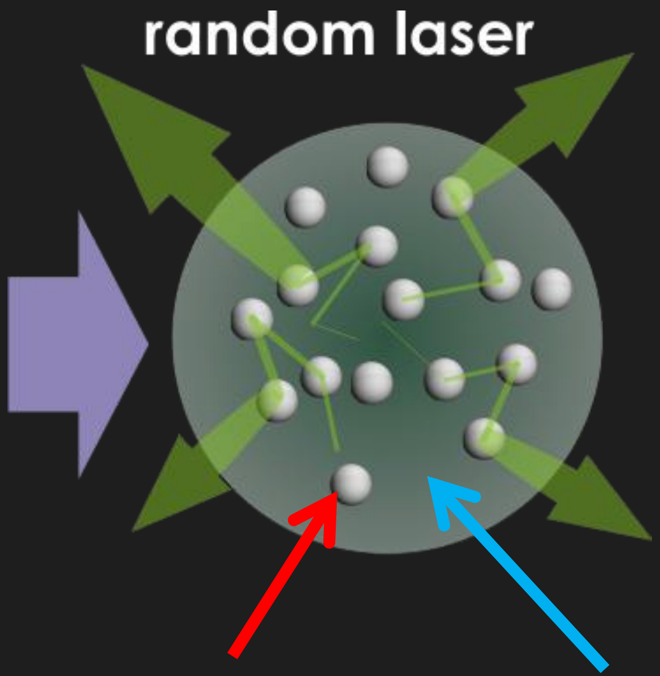
Multiple Scattering + Gain ➔ Random Laser

Letokhov, V.S. *Generation of light by a scattering medium with negative resonance absorption*. *Zh. Eksp. Teor. Fiz.* ***53, 1442–1447 (1967); Sov. Phys. JETP 26, 835–840 (1968).***

Lawandy, N. M., Balachandran, R. M., Gomes, A. S. L. & Sauvain, E. *Laser action in strongly scattering media*. *Nature* ***368, 436–438 (1994).***

Random Lasing

Multiple Scattering + Gain → Random Laser

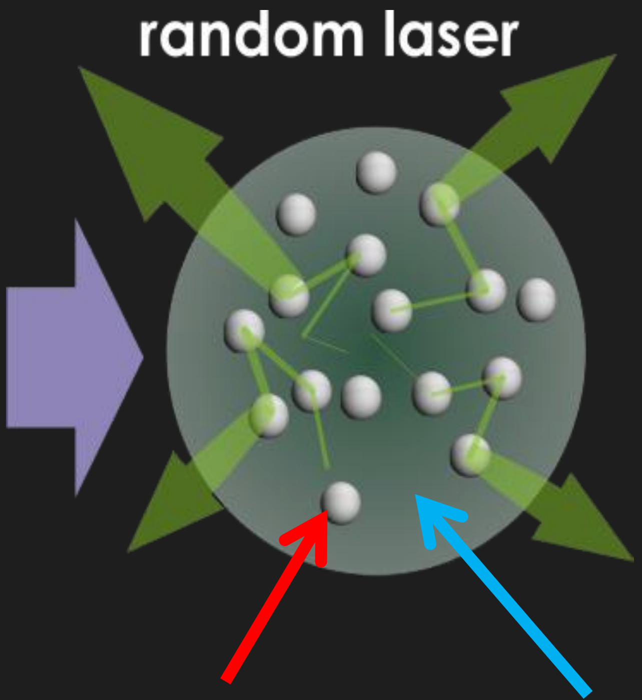


Scatterers ≠ Gain Medium

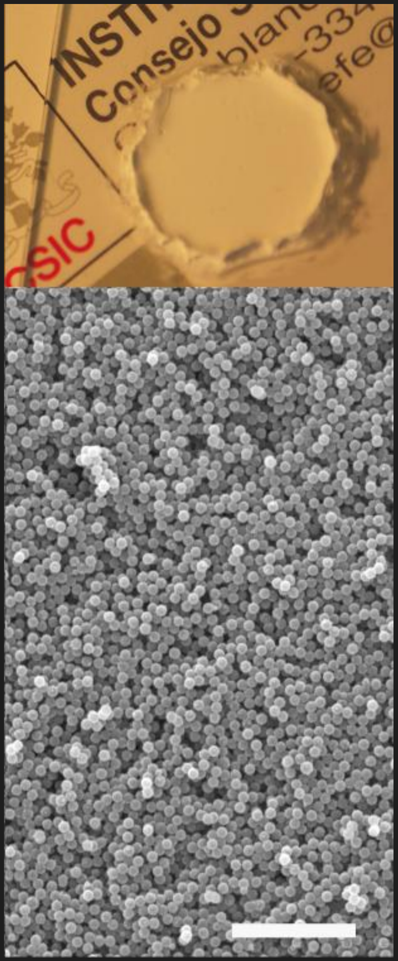
Random Lasing

… Not necessarily “bad” . Permits separate design of each component:

For instance Lasing in (dye doped) photonic glasess.



Scatterers ≠ Gain Medium



Random Lasing

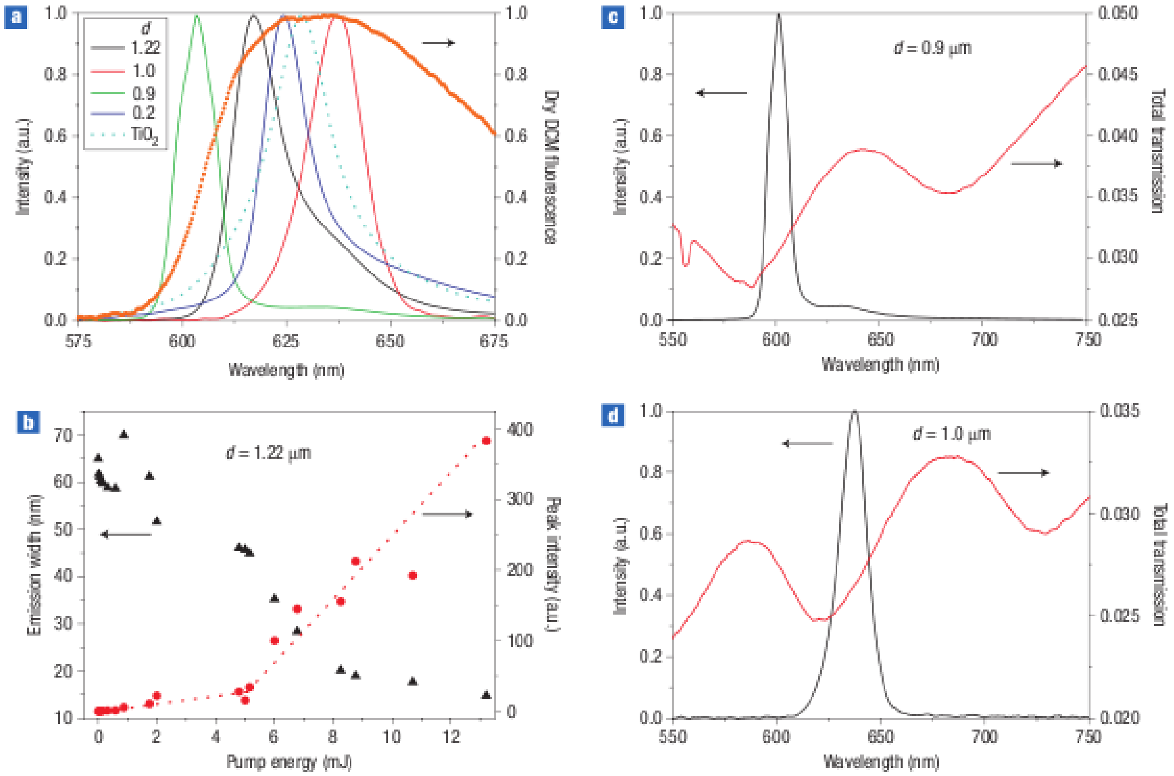


Figure 2 Random lasing action of photonic glasses. a, Random laser emission for photonic glasses with different sphere diameters compared with the pure dry-dye fluorescence (indicated by the arrow) and a reference sample made with TiO_2 powder doped with dry DCM (dotted cyan curve). The pump energy for all samples is ~14 mJ. **b**, Characteristic plot for the random laser ($d = 1.22$ μm), which highlights the threshold around 5 mJ of pump energy. **c**,**d**, Emission intensity and total transmission for photonic glasses with $d = 0.9$ μm and 1.0 μm, respectively. Lasing occurs close to the transmission minimum.

S. Gottardo, et al. “Resonance-driven random lasing”. Nat. Phot. **2**, 429 (2998)

Cold Atom Lasers (with cavity)

Cold: Avoid Doppler Effect. All the entities in the system present resonances at the same frequency

Cold Atom Lasers (with cavity)

Atom: All the entities behave in the same way (again, same resonances). There exists a bunch of experimental procedures to make atoms behave in different ways: As simple two level quantum systems for instance.

Cold Atom Lasers (with cavity)

Laser: We want to understand them!

Cold Atom Lasers (with cavity)

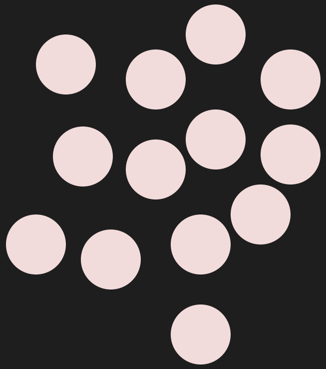
Several examples:

Depending on the pumping geometry, intensity.

Several types of cold atom lasing can be achieved.

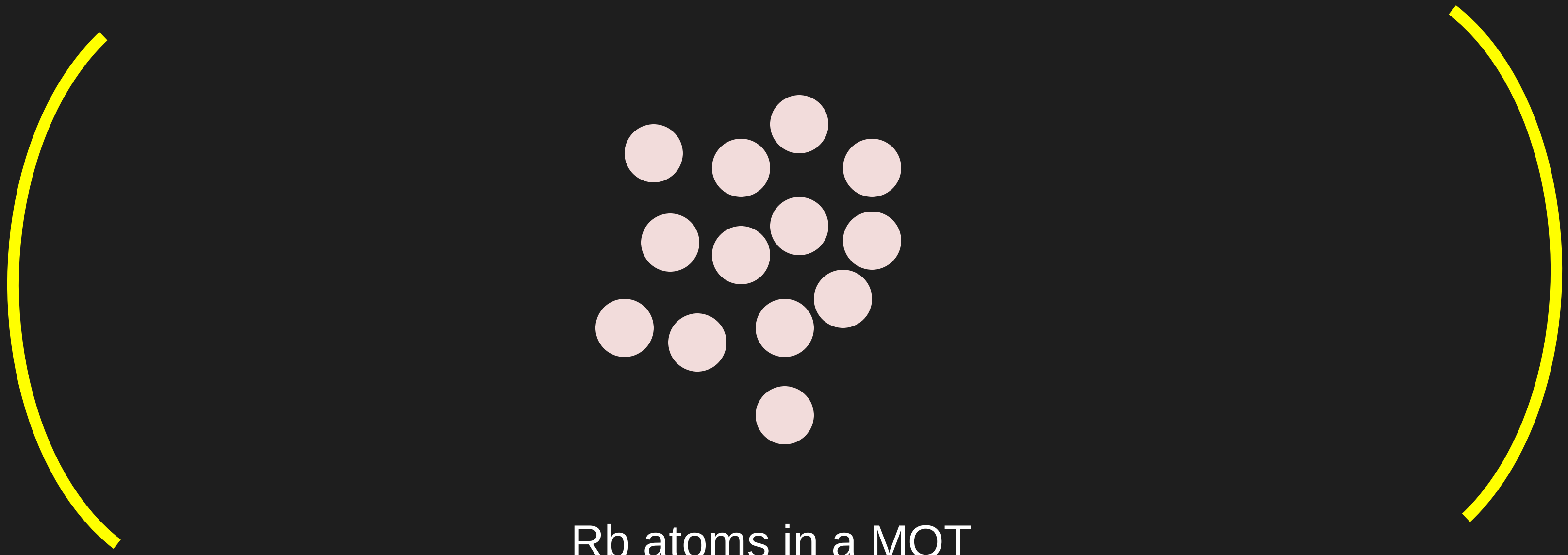
“Mechanisms for Lasing with Cold Atoms as the Gain Medium”
W. Guerin et al. PRL **101,** 093002 (2008).

Cold Atom Lasers (with cavity)



Rb atoms in a MOT

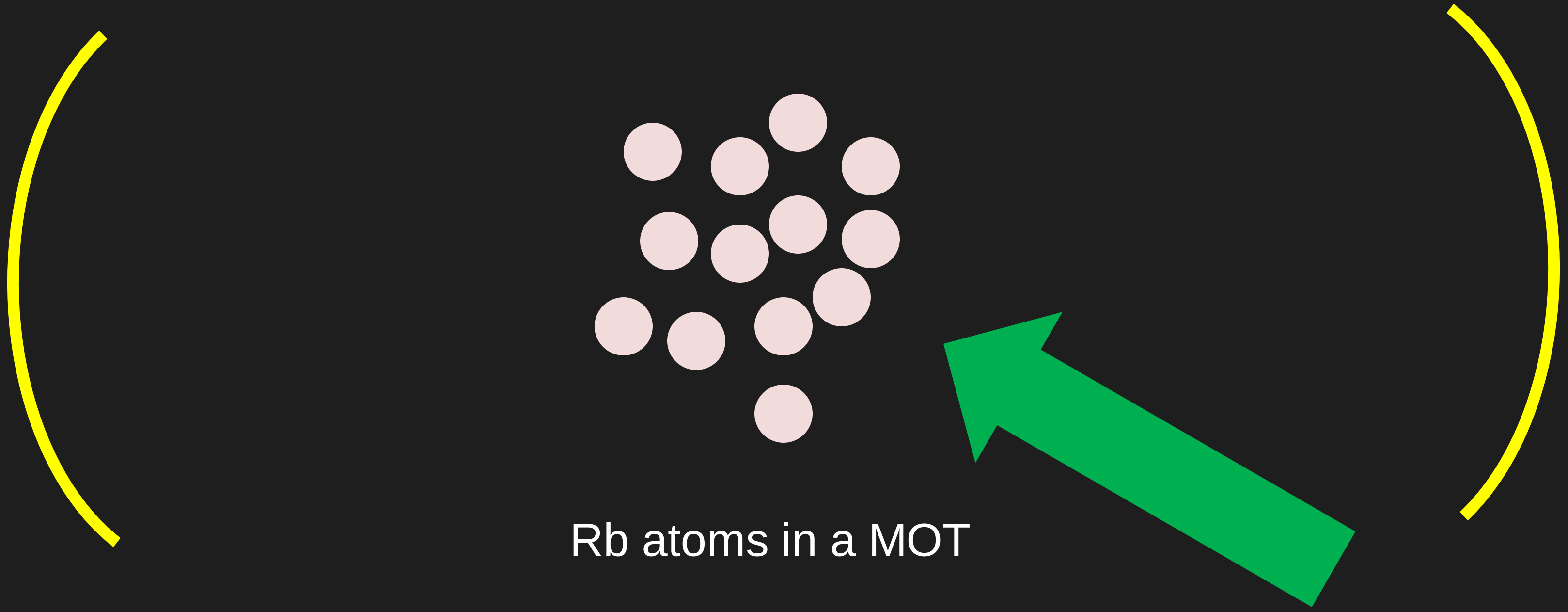
Cold Atom Lasers (with cavity)



Rb atoms in a MOT

Mirrors

Cold Atom Lasers (with cavity)



Rb atoms in a MOT

Mirrors

Pumping beam

Cold Atom Lasers (with cavity)

Several examples:
  Depending on the pumping geometry, intensity.
Several types of cold atom lasing can be achieved.

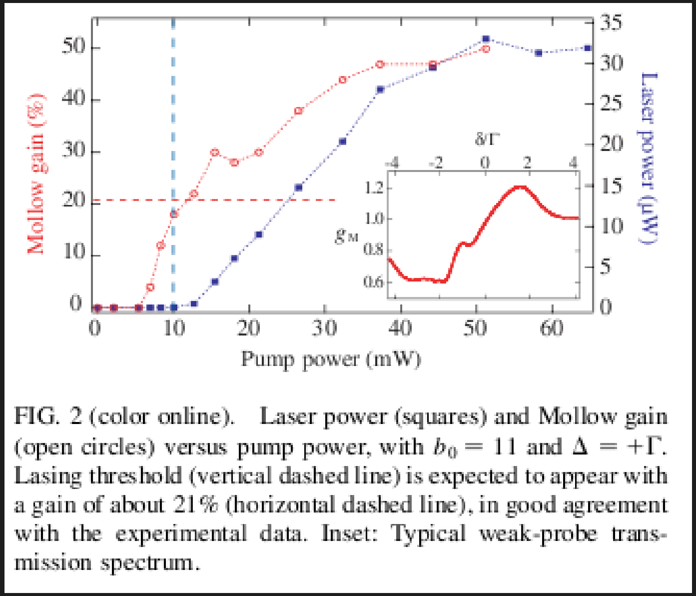
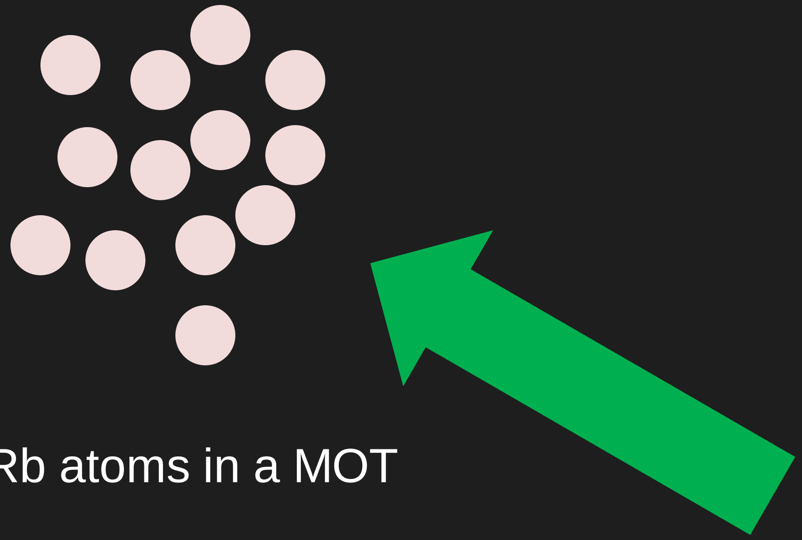


FIG. 2 (color online). Laser power (squares) and Mollow gain (open circles) versus pump power, with $b_0 = 11$ and $\Delta = +\Gamma$. Lasing threshold (vertical dashed line) is expected to appear with a gain of about 21% (horizontal dashed line), in good agreement with the experimental data. Inset: Typical weak-probe transmission spectrum.

Mollow gain: Low pump detuning

“Mechanisms for Lasing with Cold Atoms as the Gain Medium”
W. Guerin et al. PRL **101,** 093002 (2008).

Cold Atom Lasers without cavity?
Cold Atom Random Laser:



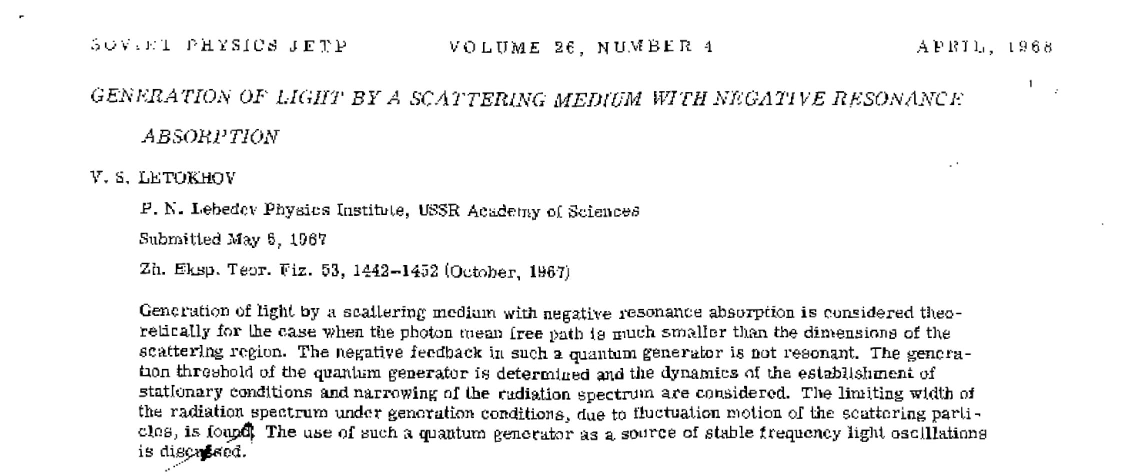
Rb atoms in a MOT

Pumping beam

Outline

- Letokhov’s theory: the photonic bomb
- Mollow gain
- The Cold atom cloud
- The Cold atom RANDOM LASER
- Conclusions

Letokhov’s theory: the photonic bomb



SOVIET PHYSICS JETP VOLUME 26, NUMBER 4 APRIL, 1968

GENERATION OF LIGHT BY A SCATTERING MEDIUM WITH NEGATIVE RESONANCE ABSORPTION

V. S. LETOKHOV

P. N. Lebedev Physics Institute, USSR Academy of Sciences

Submitted May 6, 1967

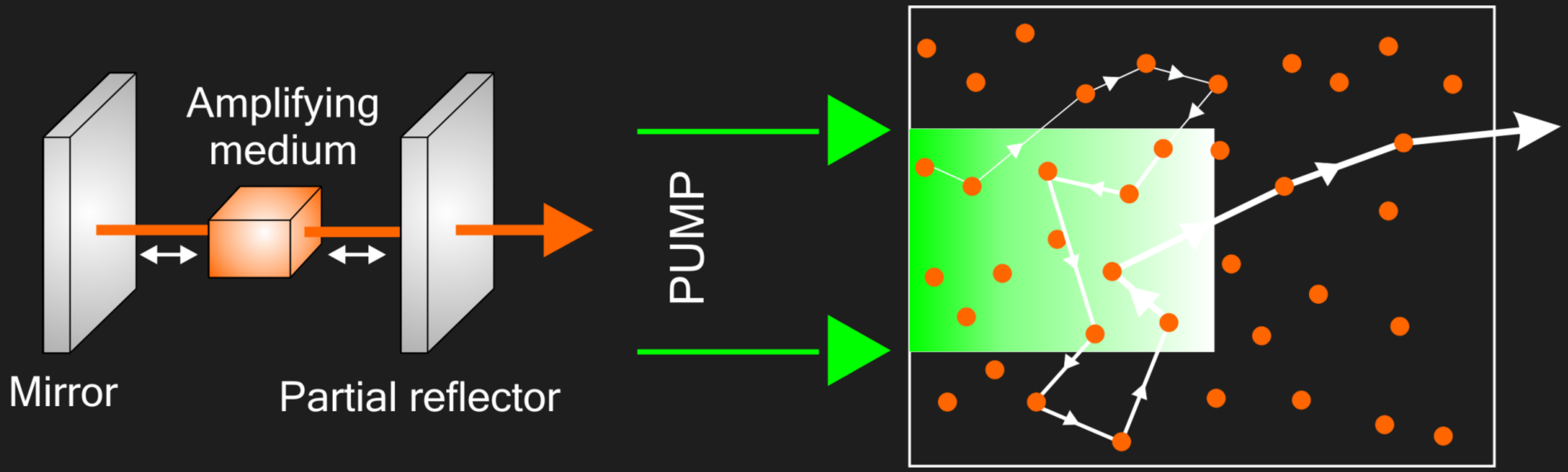
Zh. Eksp. Teor. Fiz. 53, 1442–1452 (October, 1967)

Generation of light by a scattering medium with negative resonance absorption is considered theoretically for the case when the photon mean free path is much smaller than the dimensions of the scattering region. The negative feedback in such a quantum generator is not resonant. The generation threshold of the quantum generator is determined and the dynamics of the establishment of stationary conditions and narrowing of the radiation spectrum are considered. The limiting width of the radiation spectrum under generation conditions, due to fluctuation motion of the scattering particles, is found. The use of such a quantum generator as a source of stable frequency light oscillations is discussed.

V. S. Letokhov, Sov. Phys. JETP 26, 835 (1968).

Letokhov’s theory: the photonic bomb

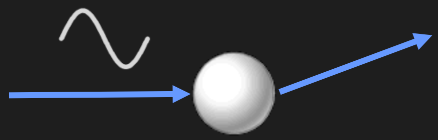
Diffusion + gain



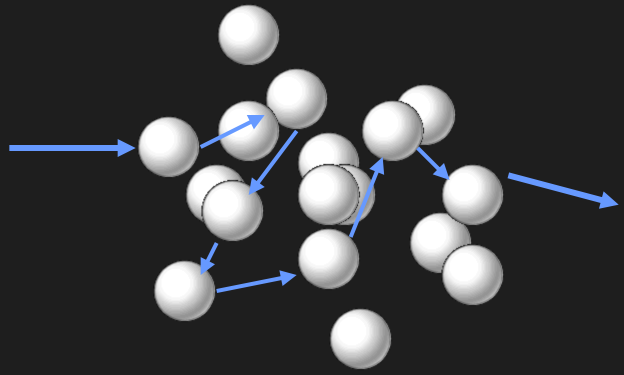
If gain > scattering lossess ➔ laser emission

Letokhov’s theory: the photonic bomb

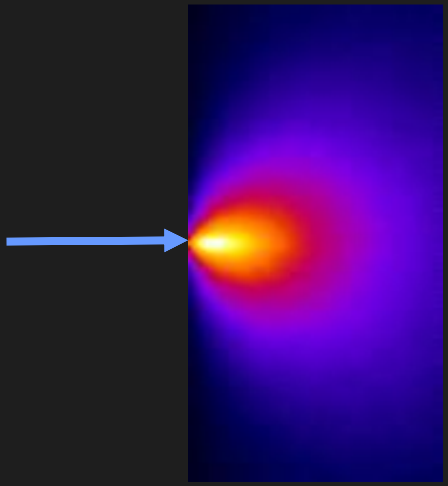
Diffusion



single scattering



multiple scattering



Light diffusion

$$\frac{\partial \Phi_\omega(\mathbf{r},t)}{\partial t} = D \triangle \Phi_\omega(\mathbf{r},t) - \ell_a^{-1} c \Phi_\omega(\mathbf{r},t)$$

Letokhov’s theory: the photonic bomb

Diffusion

$$\frac{\partial \Phi_\omega(\mathbf{r},t)}{\partial t} = D \triangle \Phi_\omega(\mathbf{r},t) - \ell_a^{-1} c \Phi_\omega(\mathbf{r},t)$$

$\Phi_\omega(\mathbf{r},t)$	spectral power density
ℓ_a	absorption length
D	Diffusion constant $D = \frac{1}{3} c \ell_t$
c	Transport (energy) velocity

Letokhov’s theory: the photonic bomb

Diffusion: modal expansion

$$\frac{\partial \Phi_\omega(\mathbf{r},t)}{\partial t} = D\triangle\Phi_\omega(\mathbf{r},t) - \ell_a^{-1} c\Phi_\omega(\mathbf{r},t)$$

$$\Phi_\omega(\mathbf{r},t) = \sum_n a_n \psi_n(\mathbf{r}) \exp\left[-\left(DB_n^2 + \ell_a^{-1}c\right)t\right]$$

$$\left[\triangle + B_n^2\right]\psi_n(\mathbf{r}) = 0$$

Letokhov’s theory: the photonic bomb

Diffusion: modal expansion

$$\frac{\partial \Phi_\omega(\mathbf{r},t)}{\partial t} = D \triangle \Phi_\omega(\mathbf{r},t) - \ell_a^{-1} c \Phi_\omega(\mathbf{r},t)$$

$$\Phi_\omega(\mathbf{r},t) = \sum_n a_n \psi_n(\mathbf{r}) \exp\left[-\left(DB_n^2 + \ell_a^{-1}c\right)t\right]$$

$$DB_1^2 + \ell_a^{-1}c < 0$$

The intensity grows exponentially with time

Letokhov's theory: the photonic bomb

Diffusion: lasing condition $DB_1^2 + \ell_a^{-1} c < 0$

B_1 depends on the **geometry**

$$B_1 = \frac{\pi\beta}{L_{eff}}$$

β=1 slab

β=2 sphere

$$L_{eff}^{(slab)} = L\left(1 + \frac{2\zeta\ell_s}{L}\right) \equiv L\eta^{(slab)}$$

$$L_{eff}^{(sphere)} = L\left(1 + \frac{2\zeta\ell_s}{L + 2\zeta\ell_s}\right) \equiv L\eta^{(sphere)}$$

$$\zeta \simeq 0.71$$

Letokhov’s theory: the photonic bomb

Diffusion: lasing condition $DB_1^2 + \ell_a^{-1} c < 0$

$$L_{eff}/\beta > \pi\sqrt{\ell_t \ell_g/3}$$

Geometry scattering gain

$$\ell_g \equiv -\ell_a$$

Letokhov's theory: the photonic bomb

Diffusion + SMALL (Rayleigh) scatterers

Scattering and absorption described by polarizability α

$$\ell_a^{-1} = -\ell_g^{-1} = \rho k_0 \left[\mathrm{Im}\left(\alpha\right) - \frac{k_0^3}{6\pi} \left|\alpha\right|^2 \right]$$

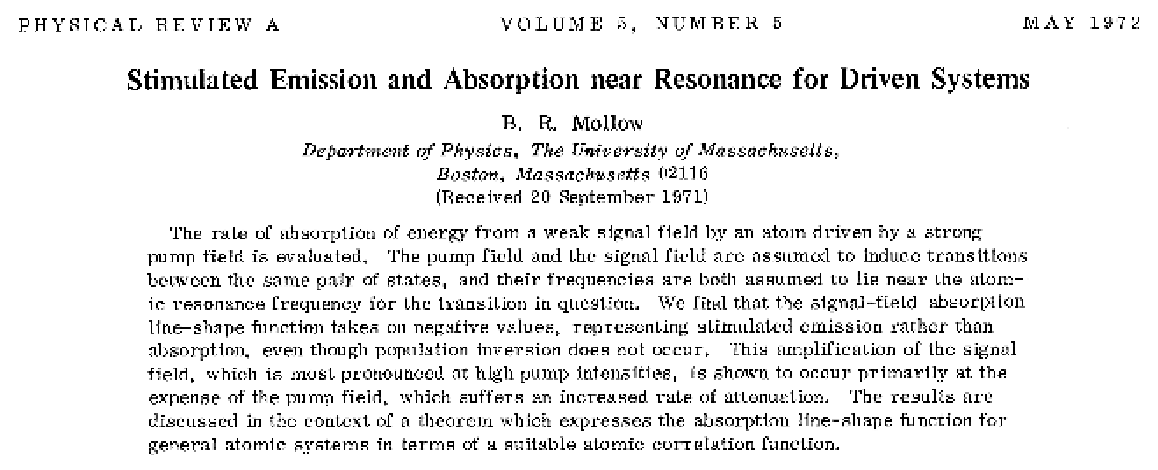
$$\ell_t \simeq \ell_s = \left(\rho \sigma_s\right)^{-1}$$

$$\sigma_s = \frac{k_0^4}{6\pi} \left|\alpha\right|^2$$

Polarizability, density, and geometry defines laser threshold

Mollow gain

Strongly pumped two level systems can present gain



PHYSICAL REVIEW A VOLUME 5, NUMBER 5 MAY 1972

Stimulated Emission and Absorption near Resonance for Driven Systems

B. R. Mollow
Department of Physics, The University of Massachusetts,
Boston, Massachusetts 02116

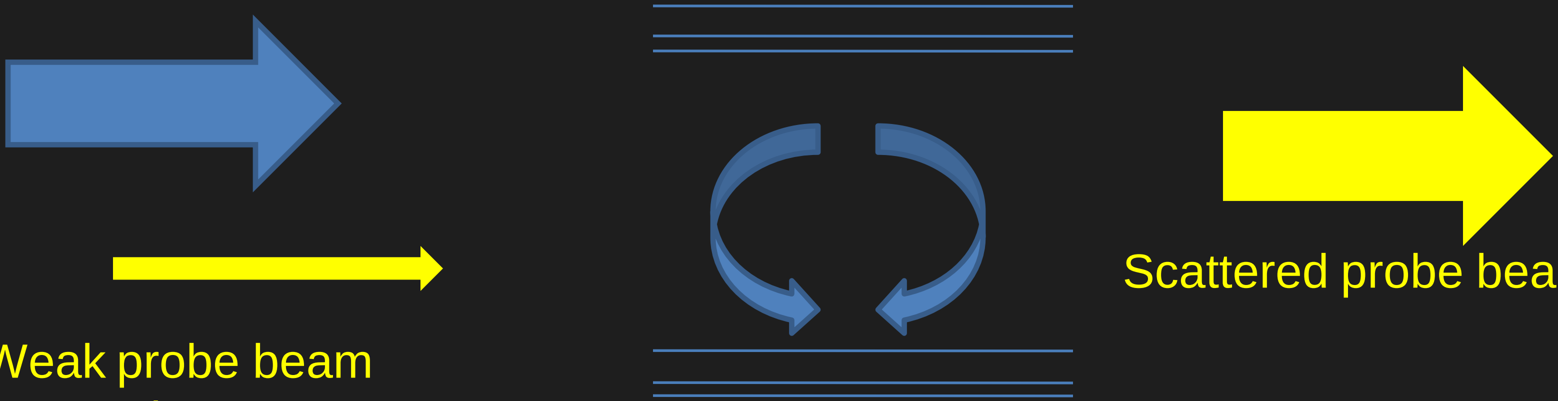
(Received 20 September 1971)

The rate of absorption of energy from a weak signal field by an atom driven by a strong pump field is evaluated. The pump field and the signal field are assumed to induce transitions between the same pair of states, and their frequencies are both assumed to lie near the atomic resonance frequency for the transition in question. We find that the signal-field absorption line-shape function takes on negative values, representing stimulated emission rather than absorption, even though population inversion does not occur. This amplification of the signal field, which is most pronounced at high pump intensities, is shown to occur primarily at the expense of the pump field, which suffers an increased rate of attenuation. The results are discussed in the context of a theorem which expresses the absorption line-shape function for general atomic systems in terms of a suitable atomic correlation function.

B.R. Mollow PRA **5,** 2217 (1972)

Mollow gain

Strongly pumped two level systems can present gain



High intensity pump:
  Intensity ~ $|\Omega|^2$ (Rabi frequency)
  Detuning Δ

Two level system
  Resonance at ω_0
  Linewidth Γ

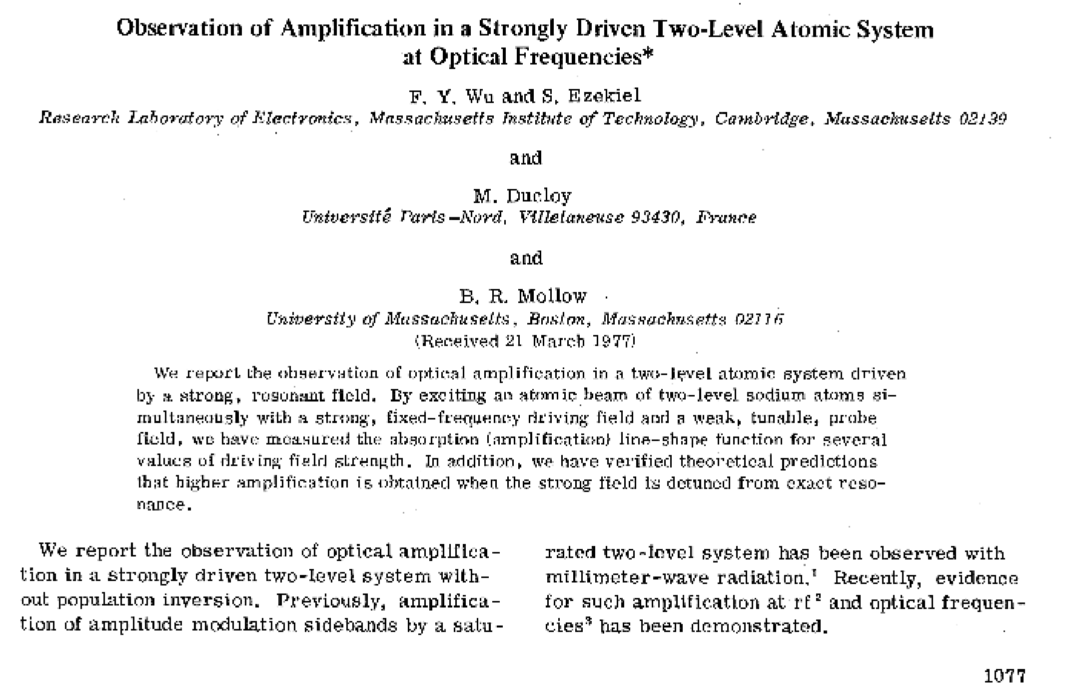
Scattered probe beam

Weak probe beam
  Detuning δ

$$\tilde{\alpha}(\delta,\Delta,\Omega) = -\frac{1}{2}\frac{1+4\Delta^2}{1+4\Delta^2+2\Omega^2}\times\frac{(\delta+i)(\delta-\Delta+i/2)-\Omega^2\delta/(2\Delta-i)}{(\delta+i)(\delta-\Delta+i/2)(\delta+\Delta+i/2)-\Omega^2(\delta+i/2)}$$

Mollow gain

Strongly pumped two level systems can present gain:
Experimentaly observed



Observation of Amplification in a Strongly Driven Two-Level Atomic System at Optical Frequencies*

F. Y. Wu and S. Ezekiel
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

M. Ducloy
Université Paris-Nord, Villetaneuse 93430, France

and

B. R. Mollow
University of Massachusetts, Boston, Massachusetts 02116

(Received 21 March 1977)

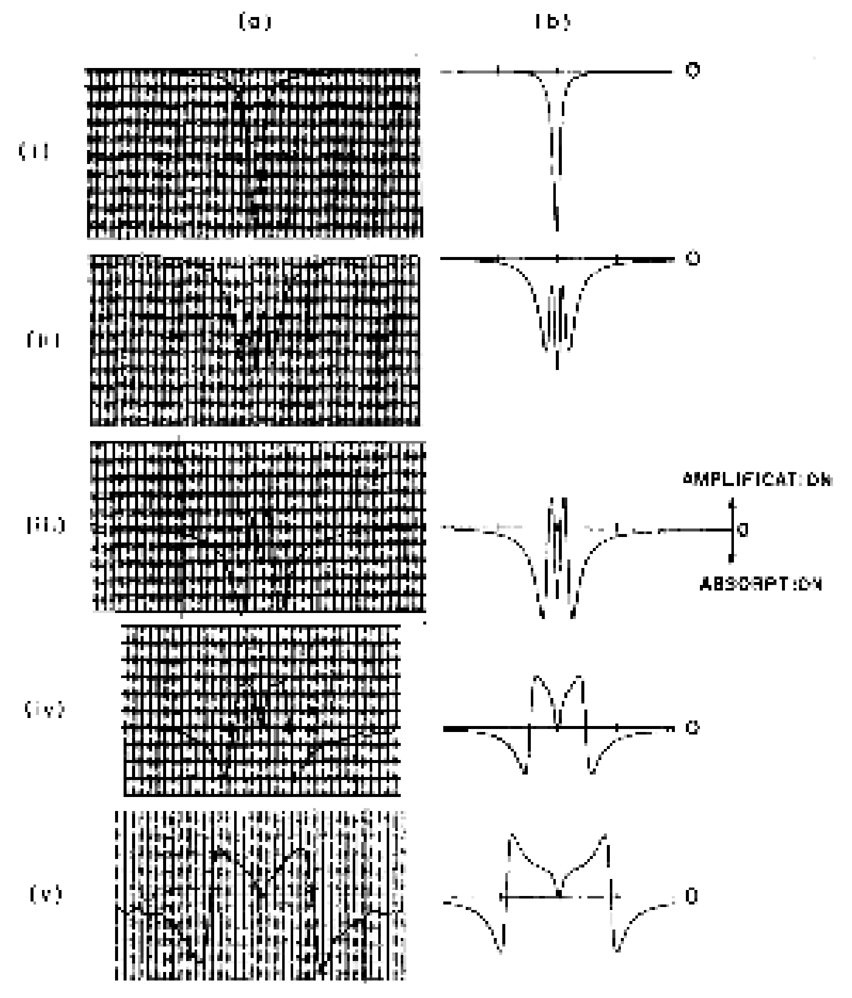
We report the observation of optical amplification in a two-level atomic system driven by a strong, resonant field. By exciting an atomic beam of two-level sodium atoms simultaneously with a strong, fixed-frequency driving field and a weak, tunable, probe field, we have measured the absorption (amplification) line-shape function for several values of driving field strength. In addition, we have verified theoretical predictions that higher amplification is obtained when the strong field is detuned from exact resonance.

We report the observation of optical amplification in a strongly driven two-level system without population inversion. Previously, amplification of amplitude modulation sidebands by a saturated two-level system has been observed with millimeter-wave radiation.[1] Recently, evidence for such amplification at rf[2] and optical frequencies[3] has been demonstrated.

F.Y. Wu et al. PRL **38,** 1077 (1977)

Mollow gain

Strongly pumped two level systems can show gain:
Experimentally observed



The Cold atom cloud

Scattering at resonance in the linear regime:
the optical thickness.

Lambert-Beer law

$$I_{sp} = I_0 e^{-\frac{L}{\ell_e}}$$

single pass outgoing intensity

Extinction length

Incoming intensity

$$\ell_e^{-1} = \rho\sigma_e$$

$$\sigma_e = k\mathrm{Im}\left[\alpha(\omega)\right]$$

The Cold atom cloud

Scattering at resonance in the linear regime:
the optical thickness.

The parameter governing the optical properties of the atomic cloud is the optical thickness

Optical thickness (b_0) = on-resonance exponent

$$b_0 \equiv L\rho k_0 \mathrm{Im}\left[\alpha(\omega_0)\right]$$

$$\tilde{\alpha}(\omega) \equiv \frac{\alpha(\omega)}{|\alpha^{linear}(\omega_0)|} = \frac{k_0^3}{6\pi}\alpha(\omega)$$

The Cold atom RANDOM LASER

Letokhov’s lasing condition:

$$\eta b_0 > \frac{\beta\pi}{\sqrt{3|\tilde{\alpha}|^2\ \left(|\tilde{\alpha}|^2 - \mathrm{Im}(\tilde{\alpha})\right)}}$$

Dimensionless polarizability

$$\tilde{\alpha}(\omega) \equiv \frac{\alpha(\omega)}{|\alpha^{linear}(\omega_0)|} = \frac{k_0^3}{6\pi}\alpha(\omega)$$

The Cold atom RANDOM LASER

consequences:

$$\eta b_0 > \frac{\beta\pi}{\sqrt{3|\tilde{\alpha}|^2\ \left(|\tilde{\alpha}|^2 - \mathrm{Im}(\tilde{\alpha})\right)}}$$

$$|\tilde{\alpha}|^2 - \mathrm{Im}(\tilde{\alpha}) > 0$$

➔ If gain then threshold must exist (perhaps at very high optical thickness).

$$\mathrm{Im}(\tilde{\alpha}) < 0$$

Is NOT necessary , i.e. single pass transmission can be smaller than one.

The Cold atom RANDOM LASER

Possibilities: Can we reach the necessary optical thickness with current technology?

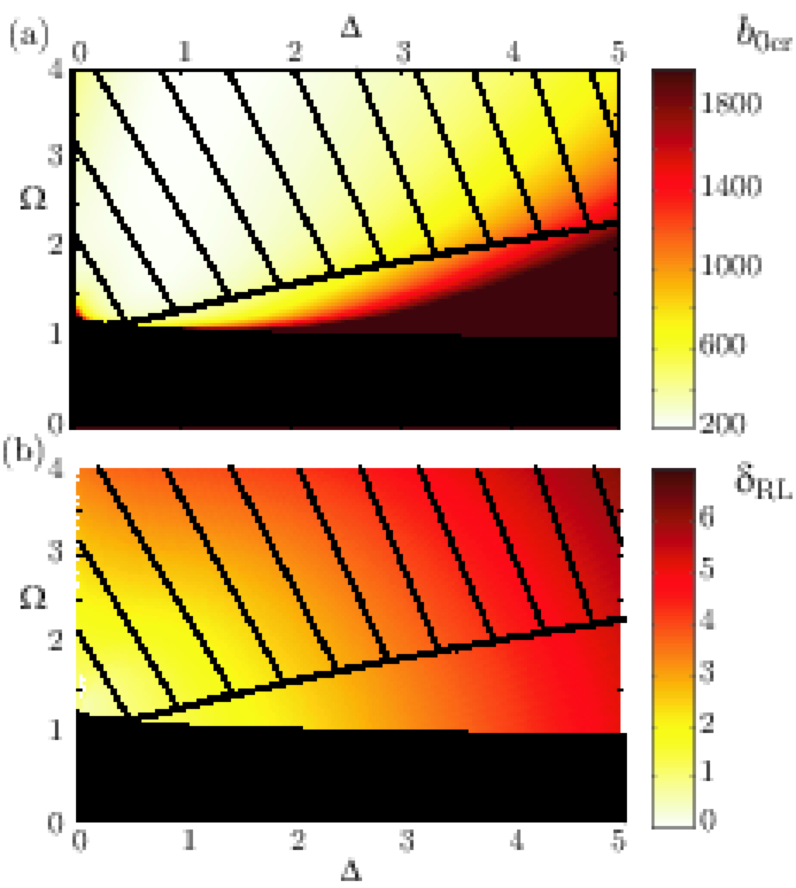
$$\eta b_0 > \frac{\beta\pi}{\sqrt{3|\tilde{\alpha}|^2\ \left(|\tilde{\alpha}|^2 - \mathrm{Im}(\tilde{\alpha})\right)}}$$

It depends on the behavior of polarizability, a case study: Mollow gain

$$\tilde{\alpha}(\delta,\Delta,\Omega) = -\frac{1}{2}\frac{1+4\Delta^2}{1+4\Delta^2+2\Omega^2}\times\frac{(\delta+i)(\delta-\Delta+i/2)-\Omega^2\delta/(2\Delta-i)}{(\delta+i)(\delta-\Delta+i/2)(\delta+\Delta+i/2)-\Omega^2(\delta+i/2)}$$

The Cold atom RANDOM LASER

Possibilities: Can we reach the necessary optical thickness with current technology?



$$\eta b_0 > \frac{\beta\pi}{\sqrt{3|\tilde{\alpha}|^2\,(|\tilde{\alpha}|^2 - \mathrm{Im}(\tilde{\alpha}))}}$$

?

But diffusion is valid if

$$L/\ell_{\mathrm{sc}} = b_0|\tilde{\alpha}|^2 \gg 1$$

We impose

$$L/\ell_{\mathrm{sc}} < 3$$

The Cold atom RANDOM LASER

Possibilities: Can we reach the necessary optical thickness with current technology?

Threshold condition using RTE instead of diffusion:

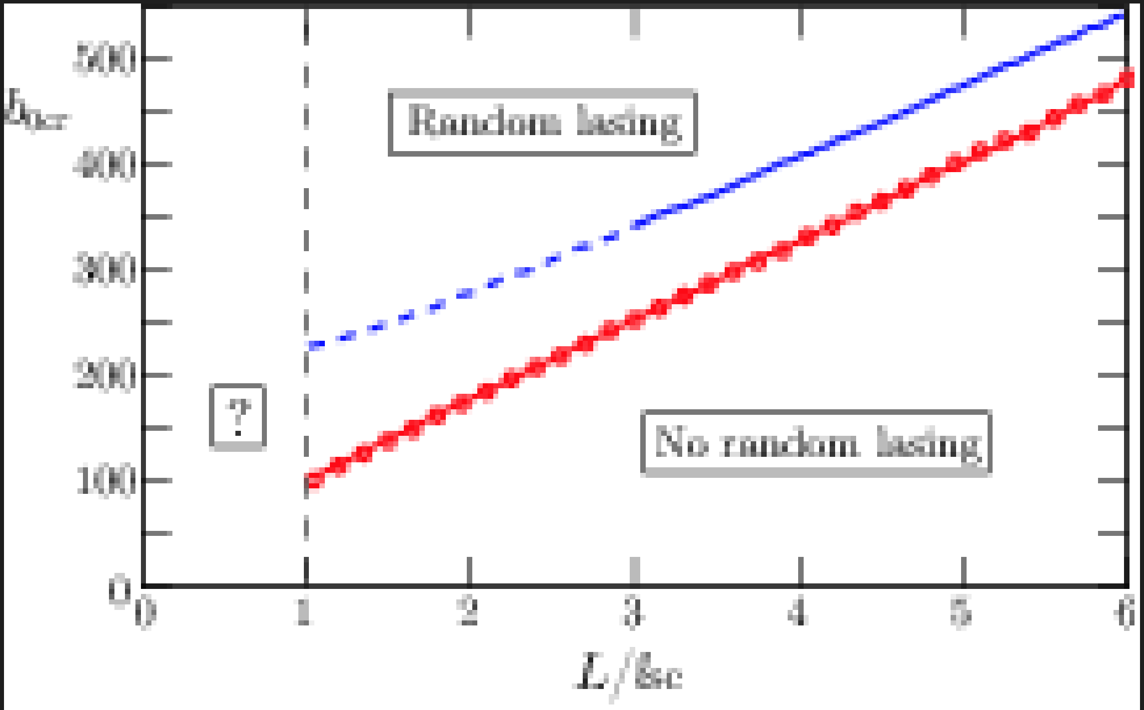
- RTE is more accurate than diffusion
- RTE It admits solutions in terms of a modal expansion.
- Only known for a slab geometry (M. D. Barrett, et al., PRL **87**, 010404, 2001)
- Has already been user to predict random laser threshold (R. Pierrat and R. Carminati, Phys. Rev. A76, 023821, 2007).

- It is valid even when $L/\ell_{\mathrm{sc}} \simeq 1$

The Cold atom RANDOM LASER

Possibilities: Can we reach the necessary optical thickness with current technology?

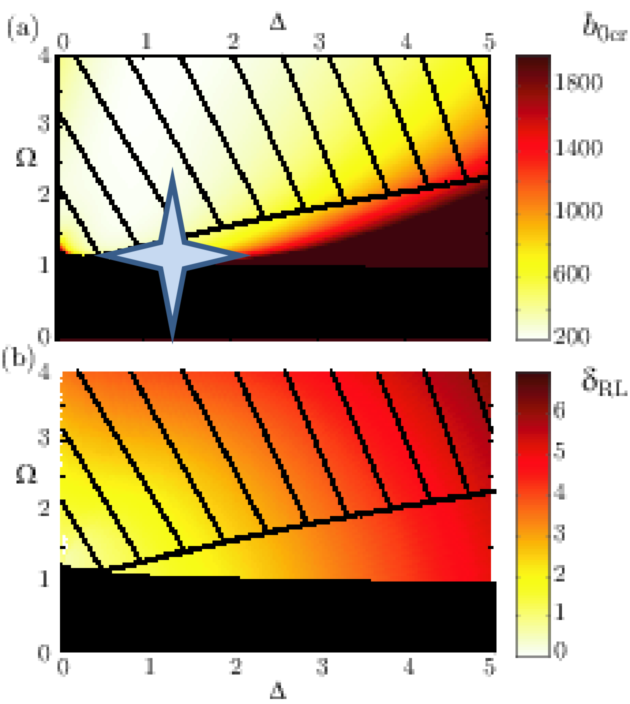
Threshold condition using RTE instead of diffusion



The Cold atom RANDOM LASER

Possibilities: Can we reach the necessary optical thickness with current technology?

Yes we can! ….. Seems to be like that.



$$b_0 \simeq 215$$

Must be reached

The Cold atom RANDOM LASER

Preliminary characterization: output power vs. pumping

- We add saturation effects also in the probe (laser) intensity.
- We get a modified Mollow polarizability
- Laser power (though its Rabi freq.) is increased until threshold is reached again: equilibrium.

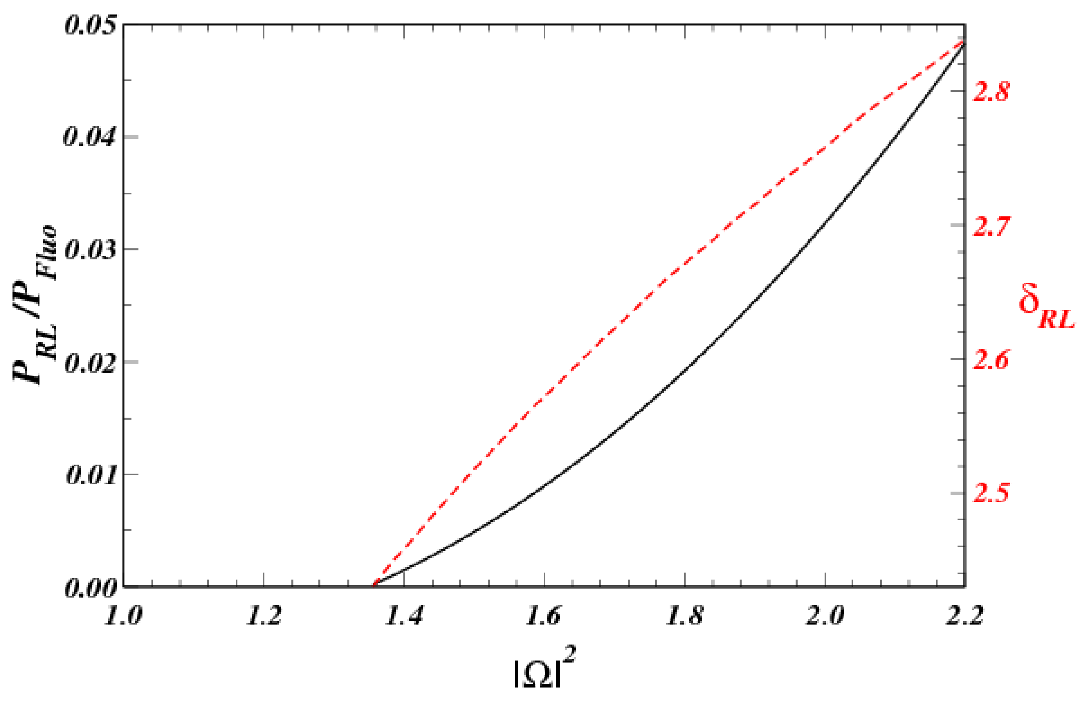
$$P_{\mathrm{Fluo}} \propto \sigma_0 |\Omega|^2 / (1 + 4\Delta^2 + 2|\Omega|^2)$$

$$P_{\mathrm{RL}}^{(\mathrm{out})} \propto \sigma_{\mathrm{g}} |\Omega_{\mathrm{RL}}|^2$$

$$\frac{P_{\mathrm{RL}}^{(\mathrm{out})}}{P_{\mathrm{Fluo}}} = \frac{|\Omega_{\mathrm{RL}}|^2}{|\Omega|^2} \left(|\tilde{\alpha}|^2 - \mathrm{Im}(\tilde{\alpha}) \right) \left(1 + 4\Delta^2 + 2|\Omega|^2 \right)$$

The Cold atom RANDOM LASER

Preliminary characterization: output power vs. pumping



b_0=650

Conclusions

- A random laser with cold atoms is possible in the Mollow scattering regime.

- A minimum optical thickness b_0~200 must be reached (currently we have b_0~20).

- Although output laser light is masked with the pump induced fluorescence, the effect should be observable

Further references:

L. S. Froufe-Pérez et al. PRL 102, 173903 (2009).
W. Guerin, et al. J. Opt. A (accepted for publication 2009)