Boltzmann and quasiballistic transport mean free paths in disordered nanowires with anisotropic scattering

J. Feilhauer[1,2], J. J. Sáenz[1] and M. Yépez[1]

[1] Condensed Matter Physics Department and Centro de Investigación en Física de la Materia Condensada (IFIMAC), Universidad Autónoma de Madrid, Fco. Tomás y Valiente 7, 28049-Madrid, Spain
[2] Institute of Electrical Engineering, Slovak Academy of Sciences, Dúbravská cesta 9, 84104-Bratislava, Slovakia
juraj.feilhauer@savba.sk

Abstract

We study an electron transport in the disordered mesoscopic nanowires with length L which contain randomly distributed anisotropic scatterers. The aim of this paper is to show that the transport mean free path ℓ in the quasiballistic regime (where $L < \ell$) can be significantly smaller than the mean free path in the diffusive regime (where $\ell << L << \xi$ and ξ is the localization length).

For simplicity we demonstrate our calculations for the case of coherent electron transport in the quasi-one-dimensional (Q1D) disordered wires made of a two-dimensional conductor but our results are more general and similar behavior should be also observable in the three-dimensional wires and electromagnetic waveguides.

In the rectangular Q1D wire with the width W the electron wave is quantized in the transversal direction and electrons at the Fermi energy occupy the discrete energy transversal states known as energy channels. To simplify the calculations we assume that the electron wavefunctions satisfy a periodic boundary conditions in the transversal direction. For the large number of channels N our calculations are independent on the choice of boundary conditions and holds also for the hard-wall boundaries. The transport properties of disordered wire are determined by the channel transmissions T_n – the probabilities that the electron impinging the disordered wire in the channel n is transmitted through the wire [1]. The transport properties depend strongly on the microscopical realization of disorder therefore it is useful to study the disorder-averaged values $<T_n>$. In the quasiballistic regime the disorder is weak and transport can be treated perturbatively (one can neglect the multiple scattering). The channel transmissions decays linearly with length as $<T_n> = 1 - L/\ell_n^{QB}$, where ℓ_n^{QB} is the quasiballistic mean free path of the channel n [2]. The total mean free path in the quasiballistic regime is then $\ell_{QB} = [\ (1/N)\ \sum 1/\ell_n^{QB}\ ]^{-1}$, where $N = 2W/\lambda_F$ is the number of occupied channels and λ_F is the Fermi wavelength [3]. In the diffusive regime the multiple scattering dominates the transport and the channel transmissions scale as $<T_n> = \ell_n^D/L$, where ℓ_n^D is the diffusive mean free path of the channel n [1]. The total diffusive mean free path reads $\ell_D = (1/N) \sum \ell_n^D$.

If the scatterers are isotropic then $\ell_n^{QB} = \ell_n^D$ which yelds $\ell_{QB} \sim \ell_D$. Does these formulas hold also for the wire with anisotropic scatterers?

To answer this question we calculate analytically the transport mean free paths in the quasiballistic (ℓ_{QB}, ℓ_n^{QB}) and diffusive (ℓ_D, ℓ_n^D) regime using the perturbative [2] and Boltzmann approach [3]. The analytical derivation can be performed in the limit of infinite number of channels N where instead of discrete index n we label the channels by the continuous angle $\theta = \mathrm{asin}(n/N)$. Then $\ell_n^{QB} \rightarrow \ell^{QB}(\theta)$ and $\ell_n^D \rightarrow \ell^D(\theta)$. The angle $\theta = 0$ corresponds to the electrons that move in parallel with the wire edges and emulate the electrons impinging perpendicularly to the sample. The anisotropic scatterers are modeled as a cylindrical potential barriers with radius a and height U which we treat in the Born approximation. Such scatterers are radially symmetric which implies that the scattering rate depends only on the angle α between the direction of the incident and outgoing electron wave. Our results are shown in the figure 1. The graphs a) and b) show the ratios $\ell^D(\theta = 0)/\ell^{QB}(\theta = 0)$ and ℓ_D/ℓ_{QB} as the functions of a/λ_F. The other graphs show the scattering rate and ratio $\ell^D(\theta)/\ell^{QB}(\theta)$ for four selected values of a/λ_F. If the scatterer is almost point-like ($a/\lambda_F << 1$) the scattering rate is isotropic (independent on α) and as expected $\ell_n^{QB} = \ell_n^D$ and $\ell_{QB} \sim \ell_D$. With the increasing radius a the anisotropy increases and the electrons are scattered mostly forwardly to the small angles. For $a/\lambda_F \sim 1$ the quasiballistic mean free path of normal incidence $\ell^{QB}(\theta = 0)$ is about three times larger than the diffusive one $\ell^D(\theta = 0)$. On the other hand the total mean free path in the quasiballistic regime ℓ_{QB} is much smaller then the diffusive one ℓ_D and this difference can be of the order of magnitude.

Our calculations confirms that the mean free paths in the wire with anisotropic scatterers (with enhanced forward scattering) can strongly depend on the wire length.

References

[1] S. Datta, *Electronic Transport in Mesoscopic Systems*, (Cambridge University Press, Cambridge, UK, 1995).
[2] P. A. Mello and S. Tomsovic, Phys. Rev. B, **42** (1992) 15963.
[3] J. Feilhauer and M. Moško, Phys. Rev. B, **83** (2011) 245328.
[4] M. Cahay, M. McLennan and S. Datta, Phys. Rev. B, **37** (1988) 10125.

Figures

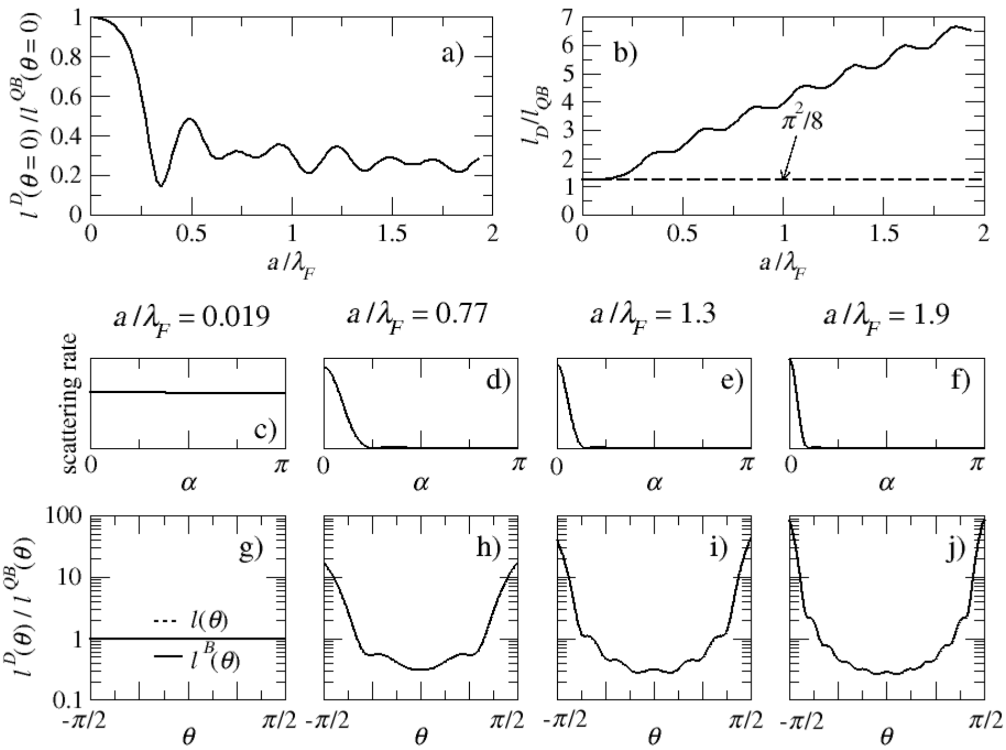

Fig. 1: a) The ratio between the mean free path of the normal incidence in the diffusive $\ell^D(\theta = 0)$ and quasiballistic $\ell^{QB}(\theta = 0)$ regime as a function of the parameter a/λ_F. b) The ratio ℓ_D/ℓ_{QB} between the transport mean free paths in the diffusive and quasiballistic regime as a function of a/λ_F. c)-f) The scattering rate as a function of the scattering angle α. g)-j) The ratio $\ell^D(\theta)/\ell^{QB}(\theta)$ between the diffusive and quasiballistic channel mean free paths as a function of the channel angle (angle of incidence) θ. The graphs c)-j) correspond to the four selected values of a/λ_F written in the legend.