Hartman effect and tunneling time in gapped graphene

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Abstract

The effect of an opening gap on the dwell time in graphene is studied. It is shown that tunneling time of massive quasiparticles through barriers in graphene as well as pure flakes is not independent of the barrier thickness and therefore Hartman effect is not observed due to tunneling of massive relativistic electrons in graphene. The numerical results reveal that the traversal time in gapped graphene is equal to the traversal time in absence of the barrier for a broad range of incident energy. It is also found that the origin of the problem of Hartman effect could be explained in terms of an average-constant behavior of the probability density of the electronic wave under the electrostatic zone. The investigations regarding the evaluation of the traversal time for a tunneling wave packet has been more subject of interest since MacColl's work concerning the evaluation of the delay time of a wave packet in transmission through a potential barrier [1]. In [2] the Hartman effect- which itself is regarded as the saturation of the delay time with barrier length - is explained in sense of the saturation of stored energy or the number of particles in the electrostatic region with increasing length of the barrier in terms of the standing waves modulation and therefore disregarding higher than light velocities for the group delay time since saturation of energy does not imply superluminal velocity for particles . However it will be shown that this argument could not be completely satisfactory as also stated in a recent work [3] in which Hartman effect has been discussed in terms of massless electrons tunneling in gapless graphene. For investigating the different aspects of traversal time associated with both pure and gapped graphene, one can use the following expression for dwell time :

$$\tau = \frac{\int_0^D dx\,|\psi_{II}|^2}{j_{in}}$$

where ψ_{II} is the electron wave function in the barrier region and $j_{in} = v_F \cos(\phi)$ is the flux of the incident Dirac fermions. Now if τ_0 indicates the the traversal time for which the electrons travel a distance *D* in graphene when the electrostatic potential is zero, one could calculate the time spent by the massless electrons with energy *E = 85 mV* , *V = 200 mV*, and *D = 100 mn* as a function of incident angle. As we see from the Fig. 1 (left) the ratio τ/τ_0 for a broad range of incident angle is approximately unity in agreement with results obtained in [3]. The result corresponding to the absence of the extra phase [4] shows (dashed curve in Fig. 1) that the evanescent propagations associated with the zero values for the dwell time begin to occur in a lower incident angles relative to the case it is included in the wave functions. It should be noted that vanishing the dwell time must not be considered as a consequence of the supernuminal velocities. It is because the transmission amplitude in the barrier zone rapidly tends to zero and therefore the time spent by the massless electrons in the barrier's zone as can be implied from the above expression for dwell time is negligible in this case. In fact it shows the time spent by a massless electron with an imaginary wave number in the electrostatic region rather the time it takes it pass through the width of the barrier and it is not something strange if by increasing *D*, no change occurs in the regime of opaque barrier. The ratio τ/τ_0 is also depicted as a function of the electron's initial energy in pure graphene for incident angle of *10* degree, the barrier's height *V = 200 mV*, and *D = 100 mn* . The results in Fig. 1 (right) indicate that the traversal time for wide range of incident energy while electrons transit the width of the barrier *D* is equal to the traversal time in the absence of the barrier meaning that there is no Hartman effect in pure graphene [3]. As it is clear in this figure near the regime of the opaque barrier (just before the dwell time vanishes) or in the other words just before the total reflection occurs two picks are observed with the different values. Now, before we turn our attention to gapped graphene, it should be noted that as the traversal time for a broad range of the initial energy is equal to case when there is no barrier it is clear that "tunneling" is indeed an appropriate term for refereeing the phenomenon of massless electrons transition across a potential barrier with energy E < V.

Calculating the dwell time of massive Dirac electrons tunneling into a barrier in a single-layer graphene with a gap opening of *30 meV* , thickness of *D = 100 nm* and potential height of *V = 200 meV* as shown in Fig.2 (right) reveals that the situation is similar to results obtained for gapless graphene meaning that Hartman effect is not observed for massive electrons tunneling, however in this case, it is observed that two extra tunneling picks corresponding to incident energy near the barrier's height in the regime of opaque barrier for pure graphene show up. Therefore one can see that the gapped barrier shows to be more transparent than a gappless one. In other words, for the energy range for which in pure graphene a complete opaque barrier stands out we have in a certain values of energy a transparent electrostatic obstacle. As it is clear from the Fig. 2 for energy E > V the dwell time is longer than the traversal time in the absence of the barrier and "transition" could be a more appropriate term for refereeing electrons transporting in this case. Another interesting feature of gapped graphene is revealed by evaluating the transmission probability as a function of the barrier thickness which turns out to be key to the independent behavior of the delay time of the thickness of the opaque barrier for a relativistic tunneling particle, i.e. the Hartman effect. As it is calculated (see Fig 3 (left)) the transmission in gapped graphene with a gap of *20 meV* oscillates independently of the barrier's width, *D*, in a range of nearly *0.2* which by increasing the energy gap this range also increases while for gapless garphene the transmission oscillates in a very narrow range of about 0.01 and one could say it is independent of the barrier's length. Now the results observed for τ/τ_0 in Fig. 1 and Fig. 2 could be explained with more details. In fact, by calculating $|\psi_{II}(x)|^2$ as a function of *x*, an arbitrary distance within the barrier, one could see that for *D = 100 nm* and in the case of barrier thickness of *D = 1200 nm* one gets the same result for probability density. As it is clear the average density for the two case with different D is the same (apart from an negligible amount which arises from the earlier-mentioned oscillatory behavior) just like the transmission probability it is independent of the value of *D* apart from a negligible oscillatory behavior for gapless graphene. Therefore, the area under the plot of $|\psi_{II}(x)|^2$ from *x = 0* to *x = D* which is $\int_0^D dx\,|\psi_{II}(x)|^2$ divided by *D*, i.e. the average of the probability density over D, is constant and independent of the barrier thickness. So it is seen that the average of the probability density barrier's width is similar to the expression for τ (regardless of the term $v_F \cos(\phi)$) which is the right expression for the dwell time.

References

[1] L. A. MacColl, Phys. Rev, **40** (1932) 621-626

[2] H. G. Winful, Phys. Rev. Lett, **91** (2003) 260401.

[3] D. Dragoman1 and M. Dragoman, J. Appl. Phys. **107**, (2010) 054306-054310.

[4] Setare M R and Jahani D, Physica B, **405** (2010)1433 .

Figures

Figure 1: Left: Dwell time as a function of incident angle for *E = 85 meV* corresponding to wave functions with a phase difference of 180 degree. Right: Dwell time as a function of energy of quasiprticles in pure graphene sheet for angle of 10 degree.

Figure 2: Left: Dwell time as a function of energy of quasiprticles in pure graphene sheet for *incident angle of 35* degree. Right: Dwell time as a function of energy in gapped graphene with *E = 20 meV*.

Figure 3: Left: Transmission as a function of barrier length for incident angle of 20 degree and E = 85meV for gapped graphene with a gap 40 meV . Right: Transmission as a function of barrier length with incident angle of 20 degree and E = 85meV for pure graphene .