Spin excitations of nanomagnets over a non-uniform magnetization ground state: Topological gauge field approach

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We developed a general approach to description of the small spin wave (SW) excitations of a non-uniform moving magnetization background in magnetic nanostructures [1]. We introduced a topological gauge vector potential which influences the SW excitation spectra over arbitrary non-uniform, slowly moving magnetization background. To describe magnetization (**M**) dynamics we used the Lagrangian Λ corresponding to the Landau-Lifshitz equation of motion of the reduced magnetization $\mathbf{m} = \mathbf{M}/M_s$:

$$\Lambda = \int d^3\mathbf{r}\lambda(\mathbf{r},t), \qquad \lambda = \mathbf{D}(\mathbf{m})\cdot\dot{\mathbf{m}} - w(\mathbf{m},\partial_\alpha\mathbf{m}), \tag{1}$$

where $\mathbf{D}(\mathbf{m}) = (M_s/\gamma)(1+\mathbf{m}\cdot\mathbf{n})^{-1}[\mathbf{n}\times\mathbf{m}]$, **n** is the unit vector in arbitrary direction, γ is the gyromagnetic ratio, the dot over symbol means derivative with respect to time (t), w is the magnetic energy density $w = A(\partial_\alpha\mathbf{m})^2 + w_m + w_H$ (α= x, y, z), $w_m = -M_s\mathbf{m}\cdot\mathbf{H}_m/2$ is the magnetostatic energy density, $w_H = -M_s\mathbf{m}\cdot\mathbf{H}$ is the Zeeman energy density, A is the exchange stiffness, $M_s = |\mathbf{M}|$, $\mathbf{H}_m$ and $\mathbf{H}$ are the magnetostatic and external fields, respectively.

We express the magnetization field **m** as a sum $\mathbf{m} = \mathbf{m}_\upsilon + \mathbf{m}_s$ of the slow moving magnetization (υ) and spin wave (s) contributions. The components of $\mathbf{m}_s$ are the simplest in a moving coordinate frame *x'y'z'*, where the axis *Oz'* (quantization axis) is directed along the instant local direction of $\mathbf{m}_\upsilon$. We perform a rotation of the initial *xyz* coordinate system to the direction of $\mathbf{m}_\upsilon$. The corresponding 3x3 rotation matrix $R(\Theta_\upsilon,\Phi_\upsilon) = \exp(i\Phi_\upsilon J_z)\exp(i\Theta_\upsilon J_y)$ is defined by the spherical angles of $\mathbf{m}_\upsilon(\Theta_\upsilon,\Phi_\upsilon)$, and the magnetization components in the *x'y'z'* frame are $\mathbf{m}'_s = R\mathbf{m}_s$, $\mathbf{m}'_\upsilon = (0,0,1)$. Here J_α are the angular momentum components for J=1 in the Cartesian basis representation. To preserve the Lagrangian (1) in the same form after the transformation $\mathbf{m}\to\mathbf{m}' = R\mathbf{m}$ we introduce covariant derivatives $(\partial_\mu - A_\mu)$ instead of ∂_μ, where A_μ is a gauge vector potential (the index μ = 0, α denotes the time and space coordinates x_μ = *t, x, y, z,* and $\partial_\mu = \partial/\partial x_\mu$). The A_μ components are transformed as $A_\mu \to A'_\mu = RA_\mu R^{-1} + \partial_\mu R\cdot R^{-1}$. The term $\partial_\mu R\cdot R^{-1}$ has sense of a topological contribution to the vector potential. We denote it as $\hat{A}_\mu = \partial_\mu R\cdot R^{-1}$ and put $A_\mu = 0$ in the laboratory coordinate system *xyz*. The gauge vector potential $\hat{A}_\mu$ represented by time and spatial derivatives of $\mathbf{m}_\upsilon$ describes a "minimal" interaction between the υ- and *s*-subsystems and can be applied to a wide class of problems related to excitation of the SW in the non-uniform magnetization ground state. There is a simple equation $\hat{A}_\mu\mathbf{m} = \mathbf{A}_\mu\times\mathbf{m}$ for arbitrary vector **m**, where $\mathbf{A}_\mu = (\sin\Phi_\upsilon\partial_\mu\Theta_\upsilon, -\cos\Phi_\upsilon\partial_\mu\Theta_\upsilon, -\partial_\mu\Phi_\upsilon)$. The Lagrangian (1) can be rewritten in the form $\Lambda = \Lambda_\upsilon + \Lambda_{sw} + \Lambda_{\rm int}$, where Λ_υ and Λ_{sw} are contributions of the slowly moving non-uniform magnetization and spin waves. An interaction term $\Lambda_{\rm int}$ described by the density $\lambda_{\rm int} = -\mathbf{D}'\cdot\hat{A}_0\mathbf{m}'_s$ corresponds to the dynamic υ – SW interaction. *I.e.*, the time-component of the potential $\hat{A}_0(\mathbf{r},t)$ plays an important role in the magnetization dynamics of typical nanostructures.

As an example, we consider spin eigenmodes excited in the vortex state cylindrical magnetic dots. The excited mainly outside the vortex core SW modes are described by integers (*n, m*), which indicate number of nodes in the dynamic magnetization along radial (*n*) and azimuthal (*m*) directions. The vortex – SW interaction is described as a consequence of the gauge field arising due to the moving vortex magnetization and represented by the components $A_{\mu}^{z} = -\partial_{\mu}\Phi_{\upsilon}$. There is the most essential contribution to the SW motion due to the variable vortex phase $\dot{\Phi}_{\upsilon} = -A_{0}^{z}$. The approach yields a giant frequency splitting (1-2 GHz) of the azimuthal spin waves with the indices *m*=+1/-1 having non-zero overlapping with the vortex gyrotropic mode (Fig. 1) as well as a renormalization of the vortex background motion due to appearance of the vortex profile distortion and a finite vortex mass of dynamical origin. The frequency splitting was measured experimentally for the modes with *n*=0 and *n*=1 by using precise broadband ferromagnetic resonance technique [2, 3]. The vortex mass is calculated as $M_{v} \approx (3/2)L/\gamma^{2}$ and is about of 10^{-20} g (*L* is the dot thickness). The renormalized vortex gyrotropic frequency can be represented in the form $\omega_{0} = 20\gamma M_{s}\beta[1-4\beta/3]/9$ at small $\beta \leq 0.2$ (β=*L/R* is the dot aspect ratio - thickness/radius (*R*)). The moving vortex core distortion and the frequency renormalization can be represented as a result of the hybridization of the gyrotropic mode with the azimuthal SW (*n*, *m*=+1/-1). The vortex core reversal can be achieved by a.c. magnetic field pumping at the frequency close to the eigenfrequencies of the azimuthal SW. Increasing amplitude of the azimuthal spin waves leads to appearance of instability of the vortex background in the form of the vortex core polarization reversal. Recent measurements of the spin eigenfrequencies in the vortex state permalloy cylindrical dots [2, 3] can be explained by the developed approach. Other dynamic magnetic nano-structures, *e.g.*, moving domain walls in nanostripes (rings) can be considered within the approach.

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References

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Figures

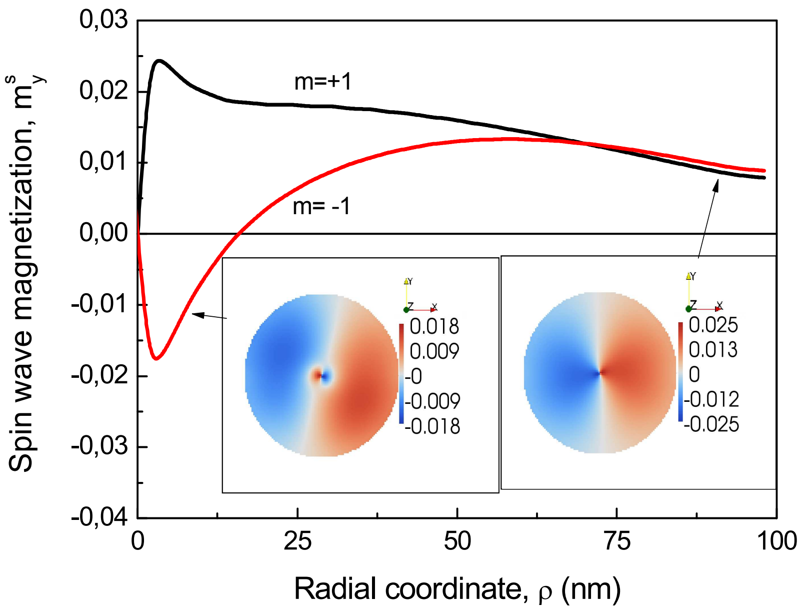

Figure 1. The calculated radial profiles of the main (the radial index n=0) azimuthal spin waves in the vortex state cylindrical dot made from permalloy. The eigenmode frequencies are $\omega_{0,+1}/2\pi$ =8.98 GHz and $\omega_{0,-1}/2\pi$ = 10.44 GHz for the SW azimuthal indices m=+1 and m=-1, respectively. Inset: snapshots of the dot dynamic in-plane m_y^s - magnetization component. The dot thickness is 10 nm, the dot radius is 100 nm, the dot saturation magnetization is M_s = 800 G, and other parameters are as in Ref. 1.