Perturbative approach for the statistical scattering of waves in disordered waveguides: closed channels contributions and the effective medium approximation

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We study the statistical properties of wave scattering in a disordered waveguide. The statistical properties of the disordered system of length L, that we call the "Building Block", are derived from a random potential model, which is constructed as a sequence of n statistically independent scattering units in the propagation direction. The scattering units consist of thin potential slices, idealized as delta potentials in the longitudinal direction of the waveguide, while the variation of the potential in the transverse direction is arbitrary. The theoretical results were obtained in the short-wave-length or weak disorder approximation (where the wave number k and the mean free path ℓ satisfy the condition $k\ell >>1$), assuming weak scattering units and by using two perturbative methods: Born series and the transition matrix method. The theoretical results were compared with numerical simulations when the waveguide supports N = 2 open channels (traveling modes) and N' = 0;1;2;3 closed channels (evanescent modes) were considered in the calculations.

Born series method predicts that the closed channels contributions are crucial for the statistics of the scattering amplitudes, while the statistic of the corresponding coefficients are insensitive to those contributions. Unfortunately, this perturbative method is only valid in the ballistic regime ($L>> \ell$), where its predictions are in good agreement with the numerical simulations; however, Born series predictions suggest that the closed channels contributions are relevant for the scattering amplitudes even beyond the ballistic regime, what is confirmed by the numerical simulations: see Fig. 1.

In order to give a more general description than Born series method, a perturbative method based on the transition matrix $\mathscr{T}$ method was performed. This method explains the intriguing contributions of the closed channels in the statistics of the scattering amplitudes, considers explicitly the multiple scattering processes and gives an excellent agreement with the numerical simulations even beyond the ballistic regime: see Fig. 2. In addition, when the waveguide admits a very large number of open channels N>>1, the transition matrix method predicts that the Building Block can be replaced by an effective potential, whose real part is too sensitive to the number of closed channels considered in the calculations. On the other hand, if the number of open channels is $N \sim 1$, it is not possible to approximated the Building Block by an effective medium, what is due to the recurrent multiplescattering.

References

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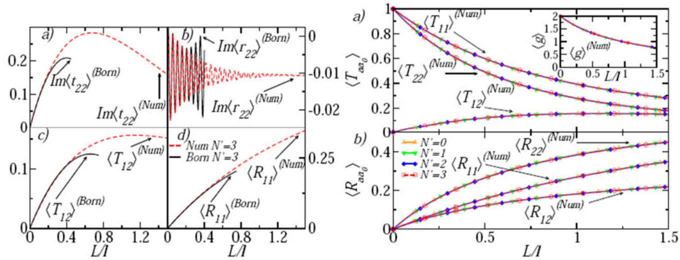

Figure 1: Left: Born series and numerical results for the expectation values of scattering amplitudes and coefficients when the waveguide supports *N*=2 open channels and *N´*=3 closed channels were considered in the calculations. Right: Numerical results for the expectation values of the coefficients.

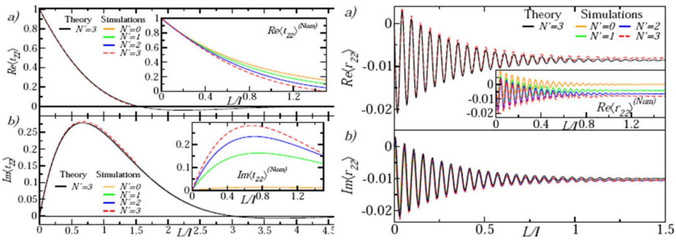

Figure 2: Transition Matrix and numerical results for the scattering amplitudes when the wave guide supports *N* = 2 open channels and *N´*=3 closed channels were considered in the calculations.