Zero bandwidth mode on a split ring resonator loaded waveguide at cutoff

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Abstract. We analyze the electromagnetic field propagation through a rectangular metallic waveguide loaded with an array of SRRs. If the split ring resonator frequency matches the cutoff frequency f_c of the hollow metallic waveguide, a flat mode is attained. To perform our study we calculate the dispersion relation using a simple transfer matrix model and confirm the results with electromagnetic simulations.

It is well known that left-handed propagation can be obtained in a structure conformed by SSRs (that can achieve negative permeability) inserted in a medium with a negative permittivity in the same region [1][2]. A square waveguide can model a one-dimensional plasma [3] with negative effective permittivity below cutoff, and therefore by introducing an array of SRRs, effective negative permeability and permittivity lead to a left-handed behavior. In this presentation we further study such configuration and obtain further interesting results in the case in which the SRR resonant frequency f_0 approaches the cutoff frequency f_c of the waveguide. We evade the effective medium explanation, due to the fact that we do not restrict ourselves to a subwavelength periodicity, and instead develop a model based on transfer matrix theory.

The structure that we analyze is shown in Fig. 1(a). The proposed circuit model for the unit cell can be seen in Fig. 1(b). Note that the model does not include electric excitation of the SRR resonance since it is small and will hardly affect the obtained response. The transmission matrix of the unit cell model is given by:

$$\mathbf{T} = \mathbf{SRR}\,(\omega, R, L, C, M) \cdot \mathbf{TL}\,(\omega, \beta, Z_c, a_z) \tag{1}$$

where $\mathbf{TL}(\omega,\beta,Z,a)$ represents the transmission matrix of a transmission line with propagation constant $\beta = \sqrt{\omega^2 \mu \varepsilon_{eff} - (\pi/a)^2}$, characteristic impedance $Z_c = \omega\mu/\beta$ and length a_z, and $\mathbf{SRR}(\omega,R,L,C,M)$ represents the transmission matrix of the SRR [4][5]. Equating $2\cos(ka_z)$ with the sum of the diagonal elements of $\mathbf{T}$ yields the dispersion relation of the periodic structure [6], given by:

$$2\cos(ka_z) = f(\omega) = 2\cos(\beta a_z) + \frac{\omega \cdot M^2}{L}\left(\frac{1}{1-\omega_0^2/\omega^2 - jR/L\omega}\right)\cdot\frac{\sin(\beta a_z)}{Z_c} \tag{2}$$

where k is the Bloch wavevector and a_z is the periodicity. Note that a propagating mode exists at a given frequency ω only if $-2 < f(\omega) < 2$. The first term of $f(\omega)$ is the term corresponding to the waveguide without rings and the second one is due to the presence of the rings. The second term is asymptotic at f_0 and responsible for the passband that appears near f_0. The width of this passband can be controlled by $\omega \cdot M^2/L$ and $\sin(\beta a_z)$ so that their value at f_0 determines the bandwidth of the mode. Interestingly, the bandwidth tends to zero when the resonance of the rings f_0 takes place at a frequency at which $\sin(\beta a_z) \to 0$. This condition can be stated as:

$$\beta\big|_{f=f_0} a_z = m\pi \tag{3}$$

In particular, if f_0 coincides with the cutoff frequency of the waveguide ($\beta = 0$), the condition (3) is achieved and a zero bandwidth mode is obtained. In order to check this result, we study a similar configuration to that presented in [3] where Marques *et. al.* demonstrate that a left-handed passband arises when the resonance frequency of the SRRs occurs below the cutoff frequency of the hollow metallic waveguide. We adjust the shape and dimensions of the SRRs to vary their resonance frequency. In particular we study in depth what occurs when the operation frequency of the SRRs is either below, above or coincident with the cutoff frequency of the waveguide. To do so we perform eigenmode electromagnetic simulations using the commercial software CST Microwave Studio™ in order to compare the numerical results with the predictions of the model. In Fig. 1(c-e) the dispersion relation described by the model and that obtained with the simulations are shown. We vary the resonance frequency of the rings around the cutoff frequency f_c of the waveguide (we call f_c to the cutoff frequency of the waveguide *without* SRRs, but considering the substrate placed inside). In the first case, when the resonance frequency of the rings (f_0) is lower than f_c a left-handed passband is observed, in agreement with [3]. When $f_0 > f_c$ the slope of the band flips its sign. At the threshold between both cases, when $f_0 \approx f_c$, a zero-bandwidth band arises. Note that in that case the original cutoff frequency of the TE_{10} mode is slightly upshifted in frequency away from the flat passband. These phenomena are predicted by the model and confirmed by the simulations. In Fig. 1(f) a plot of the group velocity at the inflection point of the passband (the point of maximum group velocity) as a function of f_0 is shown. The flip in the sign of the slope is clearly seen, as well as the point of zero group velocity. Note that, at cutoff, the waveguide shows no phase advance ($\beta = 0$), so each successive SRR "feels" the effect of the other SRRs with no phase retardation, which can be key to understanding the existence of the zero-bandwidth passband. A similar argument stands for the general condition given by Eq. (3) [although for odd values of m, the phase advance of the fields propagating in the waveguide between successive SRRs is π].

In conclusion, we have modeled a rectangular metallic waveguide loaded with an array of SRRs with a simple transfer matrix model. The calculated dispersion relation shows very close agreement with the simulations. A forward, backward or zero-bandwidth mode can be easily achieved by choosing f_0 adequately. A condition for achieving a zero-bandwidth passband is presented.

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Figures

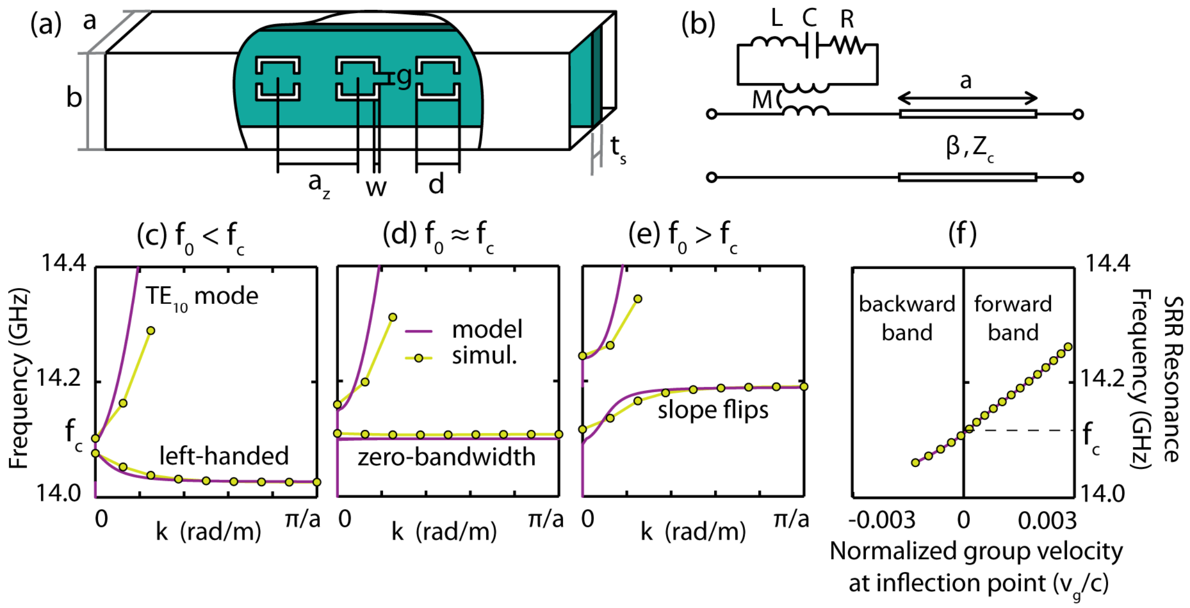

Figure 1. (a) Simulated SRR loaded waveguide. (b) Proposed circuit model for the SSR-loaded waveguide unit cell. (c-e) Modeled and simulated dispersion relations for (c) $\omega_0 < \omega_c$, (d) $\omega_0 \approx \omega_c$, and (e) $\omega_0 > \omega_c$. The parameters in the simulation (all dimensions are in mm) are: $a = b = a_z = 6$, $d = 5.2s$, $w = 0.5s$, $g = 1s$, $a_z = 10$, $t_s = 0.49$, $\varepsilon_{substrate} = 3.45^2$ and thickness of the metal $t_m=0.017$, where s if a reference feature size (c) $s = 0.48$, (d) $s = 0.4765$, (e) $s = 0.473$. The parameters used in the model are $\mu=\mu_0$, $\varepsilon_{eff} = 3.14\varepsilon_0$, $M^2/L = 4.7/\omega_0$, and (c) $f_0 = 14.026$ GHz, (d) $f_0 = 14.1$ GHz, (e) $f_0 = 14.19$ GHz. The parameters in the model such as M or ε_{eff} where chosen to fit simulations. (f) Simulated normalized group velocity at the passband inflection point plotted against the resonance frequency of the SRRs.