Nonlinearity caused by local field effects between a two level gain medium and a metamaterial

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The embedding of active media with gain to metamaterials is a main goal to overcome the problems related to the high intrinsic loss of metals at higher frequencies. A recent work [1] has shown that tailoring gain into metamaterials does not result in a simple compensation of the loss in the medium, but rather in an overall modification of the total system in a way that the gain properties of the hybrid system are affected. In this context, we study the nonlinear effects of hybrid metamaterial/gain material systems which can be essential for future applications of metamaterials. Furthermore, such systems can serve as a source of interesting new phenomena as for instance absorption cancellation near resonant transitions in analogy with the characteristics of the electromagnetically induced transparency (EIT).

In our model, we considered a two level atomic system (2LS) coupled to a metamaterial that was composed of a single layer of SRRs. The SRRs were described in the dipole approximation whereas the 2LS was treated according to a semi-classical model. The coupling between both systems is caused by the electromagnetic interaction of the local fields that penetrate the SRRs and the atomic two-level medium. The theory of such a model has been thoroughly studied in Ref. [1].

To determine the magnitude of the linear and nonlinear response of the hybrid system we described the local fields of the SRRs and the 2LS as the sum of the external field E_{ext} and the local electric fields E_{loc} at the location of the SRR and the 2LS, respectively. Thereby, it was assumed that the local electric field at the location of the 2LS was generated by the polarization fields of the SRR whereas the local electric field at the location of the SRRs was produced by the polarization fields of the 2LS. In order to take the coupling efficiency between both systems into account, we introduced a phenomenological coupling constant $\mathscr{L}$. By this means, we could describe the local fields $E_L^{(1)}$ of the 2LS and $E_L^{(2)}$ of the SRR system as

$$\begin{aligned} E_L^{(1)} &= E_{ext} + \mathscr{L} P_{SRR} \\ E_L^{(2)} &= E_{ext} + \mathscr{L} P_{12} \end{aligned} \tag{1}$$

where P_{SRR} and P_{12} are the macroscopic polarization of the SRR and the 2LS respectively, given by:

$$\begin{aligned} P_{SRR} &= N_{SRR}\mu_{SRR}\rho_{SRR} \\ P_{12} &= N_{12}\mu_{12}\rho_{12} \end{aligned} \tag{2}$$

where $N_{SRR,\,(12)}$ is the density of SRR, (atoms), $\mu_{SRR\,,(12)}$ is the dipole transition moment of the SRR (two-level system from the ground to excited state), and ρ is the slowly varying amplitude of the coherence.

From the Liouville equation for the reduced density matrix of the SRR we obtained for the equation of the SRR in the rotating wave approximation

$$\dot{\rho}_{SRR} = -i\left(\Delta_{SRR} - i\gamma_{SRR}\right)\rho_{SRR} + i\frac{\mu_{SRR}}{\hbar}E_L^{(2)}, \tag{3}$$

where $\Delta_{SRR}=\omega-\omega_{SRR}$ is the detuning of the external field frequency ω from the resonance frequency ω_{SRR} of the SRR; and γ_{SRR} is the damping constant of the SRR.

In analogy, the dynamics of two level system with ground state |1> and excited state |2> that interacts that interacts with an optical field closely tuned to the atomic resonance, can be described by the Maxwell-Bloch equations [2]

$$\begin{aligned} \dot{\rho}_{12} &= -i\left(\Delta_{12} - i\gamma_{12}\right)\rho_{12} - i\frac{\mu_{12}}{\hbar}E_L^{(1)}w \\ \dot{w} &= -\Gamma_{12}(w+1) + i\left(\rho^{*}{}_{12}\frac{\mu_{12}}{\hbar}E_L^{(1)} - \rho_{12}\frac{\mu_{12}}{\hbar}E_L^{*(1)}\right), \end{aligned} \tag{4}$$

where ρ_{12} is the amplitude of the coherence, $\Delta_{12}=\omega-\omega_{12}$ is the detuning, γ_{12} and Γ_{12} are the transversal and longitudinal damping of the 2LS, respectively. *w* is the population inversion; we assume that the equilibrium value, corresponding to the ground state, is *-1.* By inserting Eqs. (1) and (2) into Eqs. (3) and (4) we obtained a complete system from which we could calculate the effective electric susceptibility χ of the system as a function of the macroscopic polarization fields as:

$$\chi = V_{12}\frac{P_{12}(\rho_{12},w)}{\varepsilon_0 E_{ext}} + V_{SRR}\frac{P_{SRR}(\rho_{SRR},w)}{\varepsilon_0 E_{ext}}, \qquad (5)$$

where $V_{12,(SRR)}$ is the volume fraction of the gain material(SRR) in the system.

For illustrating the physical effects of the nonlinearity in the system it is convenient to represent the effective susceptibility as a power series expansion of the external electric field

$$\chi = \chi^{(1)} + \chi^{(3)}|E_{ext}|^2 + \chi^{(5)}|E_{ext}|^4 + \ldots \qquad (6)$$

to convert Eq. (5) in the form of Eq. (6) we performed a Taylor expansion of the population inversion *w* in terms of $x=|E_{ext}|^2/|E_s|^2$ where E_s is the analogous of the saturation field strength [3] of this system. Introducing this expansion in Eq. (5) we obtain the expression given by Eq.(6).

We will show that *w* changes from *-1* to other values when high external fields are applied. This change involves two nonlinear effects: the amplitude change of the susceptibility response and the modification of the resonance frequency.

In Fig. 1 we show the parameter range where the different orders of the Taylor expansion of the susceptibility are valid when there is not inversion or pumping and for same resonance frequencies.

In our presentation, we will show how nonlinear effects induce saturation effects and cascaded processes in a hybrid SRR/2LS system due to local field interactions and will discuss how the nonlinearities influence the dynamic characteristics of the composed system

In conclusion, we calculated the nonlinear electric susceptibilities for a metamaterial that was coupled to a two level gain material. Starting from the corresponding Maxwell Bloch equations we could evaluate the contributions of the linear, third and fifth-order electric susceptibility of the hybrid system to the overall nonlinear response by means of a Taylor expansion in the population inversion of the two-level gain medium. Our results for the nonlinearities in the susceptibility show the range of validity of the different approximations and the underlying physic responsible for these nonlinear effects. Finally, we show effects in the dynamic of the system when it is pumped.

References

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Figures

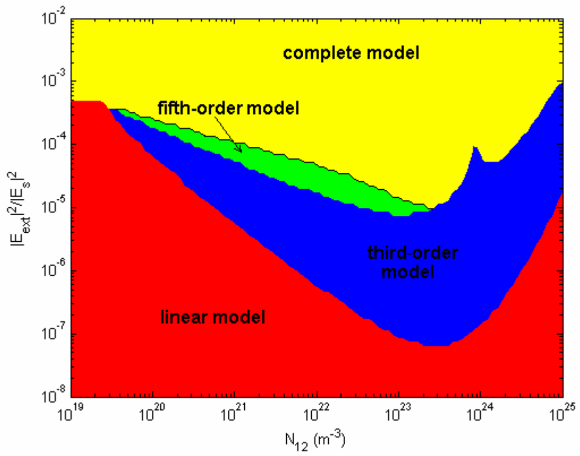

Fig. 1: (Color online) The ranges of validity of the different orders of the susceptibility in function of the dipole density N_{12} of the two level system and the parameter $x=|E_{ext}|^2/|E_s|^2$.