Monolayer and bilayer graphene in the presence of a strong magnetic field: FQHE

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Abstract

We present a cyclotron subgroup model of the fractional quantum Hall effect (FQHE), which is entirely based on the mathematical concept of braid groups, $\pi_1(\Omega)$ – Ω stands for a N-particle configuration space. In the standard quantization method, multi-particle state vectors, Ψ_N, are selected as functions from Ω into complex numbers [1]. Thus, when arguments of Ψ_N are encircling a closed path in their configuration space, then the multi-particle wave function acquires a phase equal to a one-dimensional unitary representation (1DUR) of the corresponding braid from $\pi_1(\Omega)$. The braid group, hence, defines the quantum statistics of particles. We argue that, in the presence of a magnetic field, classical trajectories representing elements of $\pi_1(\Omega)$ are of a cyclotron orbit type. In 2D multi-particle systems this leads to the reduction of a full braid group (not all braids are accessible). Additionally, only if this restricted version has a group form, then the particle statistics – unavoidable for any correlated Hall-like state - can be determined. Here we identify all possible cyclotron subgroups of $\pi_1(\Omega)$ and we use them to construct hierarchies of filling factors for monolayer graphene samples. We notice that, similarly to the conventional 2DEG, the lowest Landau level (LLL) pyramid of fractions is a generalized version of the composite fermion hierarchy [2]. Thus, our model provides a topological explanation of the Jain's approach - a pair of pinned flux quanta is a convenient model of an additional loop appearing in the $\pi_1(\Omega)$ generators definition. It should be emphasized that a nonzero Berry phase causes the graphene LLL to be placed exactly in the Dirac point – where the valence band meets the conduction band in a gapless energy spectrum. The lowest Landau level is, thus, equally shared between free electrons and free holes. Since it is natural to define the filling factor in terms of an electronic density measured from the charge neutrality point, hence, ν is counted with respect to the bottom of a conduction band (a third spin-valley branch) and not the LLL as in typical semiconductors.

The area encircled by a classical cyclotron orbit, which represents a $\pi_1(\Omega)$ element, is proportional to the bare kinetic energy of electrons. For this reason – despite that graphene Landau levels are not distributed equidistantly and that an effective mass of (Dirac) particles is equal to zero – we could arrive at a Jain-like hierarchy in the LLL. Simultaneously, however, we expect that in higher Landau levels the quantum statistics and, so, the braid group describing the system can be determined for fillings, which are not necessarily counterparts of those from the LLL. Here we propose the hierarchies and we emphasize the emergence of a novel, extremely robust Hall effect (the single-loop FQHE). These results agrees with recent experiments [3] (also for conventional 2DEG [4]) and clarify the ineffectiveness of a quasiparticles approach within higher Landau bands.

Finally, we implement the cyclotron subgroup model to the case of bilayer graphene structures. We explain how an additional surface (arising from a supplementary layer of carbon atoms) can results in a surprising form of the basic set – where $\nu=1/2$ is a most prominent incompressible state [5]. What is especially important the statistics of particles for even-denominator fractions in graphene bilayers remains antisymmetric (a power in a Jastrow-like polynomial remains odd). This is due to the fact that one of loops, which appear in a non-trivial cyclotron trajectory (a representative of a double generator of $\pi_1(\Omega)$), is utilizing a surface supplied by an additional layer. As a result it cannot contribute to the effective cyclotron area enlargement process, but it still needs to be taken into account while analyzing the quantum statistics of particles.

The support from the NCN Project UMO-2011/02/A/ST3/00116 is acknowledged.

References

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