Testing the index theorem for graphene and C60 molecules

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Abstract

To introduce a positive Gaussian curvature into a planar sheet of graphene, one cuts a segment with opening angle $\pi/3$ from the hexagonal lattice and attaches the open ends to one another. This amounts to introducing a single pentagon into the lattice which acts as a topological defect. To obtain a fullerene C60 molecule, one needs to introduce 12 pentagons.

Mathematically, the curvature can be described by two gauge fields: One gauge field is used to describe the curved topology, which can be represented by magnetic monopoles, and the other gauge field is used to describe the mixing of the two sublattices. The topological index of the Dirac operator on the curved manifold can then be calculated using the Euler characteristic of the manifold. The analytical index of the Dirac operator is the difference of the number of its zero energy states at the two Fermi points.

The Atiyah-Singer index theorem gives a connection between the topological index and the analytical index: It states that they are the same [1]. We propose to test the index theorem, using experimental results obtained from microwave billiards.

Planar graphene has been simulated successfully with a microwave billard in Refs. [2, 3]. The plan is to apply a similar methodology to the case of a C60 fullerene molecule. The billard, in this case, is a microwave resonator whose cavity has the form of the 60 carbon atoms. In collaboration we will analyze the excitation spectrum of the fullerene molecule and verify that the index theorem is fulfilled in these experiments.

References

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