TWO-DIMENSIONAL CHARGE RELAXATION IN GRAPHENE: GENERALIZED TELEGRAPH EQUATIONS AND PSEUDO-RELATIVISTIC INVARIANCE

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Electrostatically doped graphene is not an intrinsic system and necessarily provided by the isolated conductive gate which controls electric neutrality and the Fermi energy position in graphene. The energy accompanying the current can be stored in associated magnetic field and as kinetic energy of flowing carriers. We will consider here the case $v_0 / L \le \omega << c / L$ (*) that allows neglecting the magnetic inductance in comparison to kinetic inductance describing mechanical inertia. These inequalities enable us to set the light velocity equal to infinity neglecting in such way the photon retardation and magnetic inductance in comparison with plasmon retardation and kinetic inductance associated with mechanical motion. The non quasi-static operation is generally described by the electrodynamics and transport equations. Neglecting recombination and generation in the channel, the two first moments of the semi-classical Boltzmann transport equation are written as [1]

$$\frac{\partial \mathbf{j}(\mathbf{r},t)}{\partial t} + \frac{\mathbf{j}(\mathbf{r},t)}{\tau} = \frac{1}{L_K}\nabla_{\mathbf{r}}\left[-\frac{\mu(\mathbf{r},t)}{e}\right], \qquad \nabla_{\mathbf{r}}\mathbf{j}(\mathbf{r},t) + \frac{\partial \rho(\mathbf{r},t)}{\partial t} = 0\,, \tag{1}$$

where $\mathbf{j}(\mathbf{r},t)$ and $\rho(\mathbf{r},t)$ are 2D current and charge densities, $\mu(\mathbf{r},t) = \zeta(\mathbf{r},t) - e\varphi(\mathbf{r},t)$ is local value of electrochemical potential (ζ and φ are chemical and electric potentials), σ_0 is 2D conductivity, and kinetic inductance associated with mechanical carriers' motion is written for graphene as

$$L_K = \frac{\tau}{\sigma_0} = \frac{\tau}{e\mu_0 n_S} = \frac{h}{2e^2}\frac{1}{k_F v_0} \tag{2}$$

For quasi-static case we have equations for incompressible fluid $\nabla_{\mathbf{r}}\mathbf{j}(\mathbf{r},t) = 0$, $\mathbf{j} = -\sigma_0 \nabla_{\mathbf{r}}\left(\mu(\mathbf{r})/e\right)$. For uniform conductivity the electrochemical potential distribution is obeyed to the Laplace equation $\nabla_{\mathbf{r}}^2 \mu(\mathbf{r}) = 0$. For long-range charge non-uniformities in graphene the ratio of the diffusion to the drift current components (assumed to be position-independent for a given electric biases) of the total current density $\mathbf{j} = \mathbf{j}_{DR} + \mathbf{j}_{DIFF} = (1+\kappa)\mathbf{j}_{DR}$ may be obtained using the electric neutrality condition of gated graphene structure [2], which assumed to be instantaneous implying the condition (*)

$$\kappa = \frac{j_{DIFF}}{j_{DR}} = -\left(\frac{\partial \zeta}{e\partial \varphi}\right)_{V_G} = \frac{\left(\partial V_G/\partial \varphi\right)_{\zeta}}{e\left(\partial V_G/\partial \zeta\right)_{\varphi}} = \frac{C_{ox}}{C_Q} = \frac{\varepsilon_a}{\varepsilon_F}\,, \tag{3}$$

where $C_Q = 2\varepsilon_F / \pi\hbar^2 v_0^2$ is the graphene quantum capacitance, C_{ox} is the gate oxide capacitance per unit area, and the characteristic energy is introduced

$$\varepsilon_a = \frac{\pi\hbar^2 v_0^2 C_{ox}}{2e^2}\,, \tag{4}$$

which is nothing but the full electrostatic energy stored in the capacitor with the area per one carrier in gapless graphene [3]. Quasi-static current as well as charge and potential distributions are obtained analytically in Ref.[2] in a such way. The Eqs.1 can be rewritten as follows

$$\frac{\partial^2 \rho(\mathbf{r},t)}{\partial t^2} + \frac{1}{\tau}\frac{\partial \rho(\mathbf{r},t)}{\partial t} = \frac{1}{eL_K}\nabla^2 \mu(\mathbf{r},t)\,. \tag{5}$$

This is nothing but a generalized form of the telegraph equations, which could be also derived from circuit consideration for a distributed RLC transmission line. Suppose a fluctuation $\delta\varphi$ is created and we ask how the system will go back to equilibrium. Notice that for long-range non-uniformities we have $\delta\rho = C_Q \kappa \delta\varphi$, $\delta\mu = (1+\kappa)e\delta\varphi$

$$e\frac{\delta\rho}{\delta\mu} = \frac{\kappa}{1+\kappa}C_Q = \frac{C_{ox}C_Q}{C_{ox}+C_Q} = \frac{\partial e n_S}{\partial V_G} = C_{CH} \tag{6}$$

and "compressibility" is proportional to the channel capacitance. Taking this into account one can obtain a dissipative wave equation, which can be written in two equivalent forms

$$v_S^2 \nabla^2 u(\mathbf{r},t) - \partial_t^2 u(\mathbf{r},t) = \tau^{-1}\partial_t u(\mathbf{r},t)\,, \qquad D_S \nabla^2 u(\mathbf{r},t) = \partial_t u(\mathbf{r},t) + \tau\,\partial_t^2 u(\mathbf{r},t)\,, \tag{7}$$

where $u(\mathbf{r},t)$ is any of functions $\delta\varphi$, $\delta\rho$ and $\delta\mu$, the characteristic velocity of the signal propagation v_S and an effective diffusivity D_S of signal are introduced as follows

$$v_S^2 = \left(C_{CH} L_K\right)^{-1} = v_0^2\left(\kappa+1\right)/2\kappa\,, \qquad D_S \equiv v_S^2\tau = D_0\left(\kappa+1\right)/\kappa\,, \tag{8}$$

and the "signal diffusivity" coincides with carrier diffusivity $D_0 = v_0^2\tau/2$ only for low doping when $\varepsilon_a >> \varepsilon_F$ (notice the Einstein relation $\sigma_0 = C_Q D_0 = C_{CH} D_S$). For $\omega\tau << 1$ we have diffusion-like propagation with loss of the signal $\partial_t u \cong D_S \nabla^2 u$. In contrast, for $\omega\tau >> 1$ we have the non-dissipative wave equation

$$v_S^2 \nabla^2 u\left(x,t\right) = \partial_t^2 u\left(x,t\right) \tag{9}$$

with the signal propagation speed dependent on the ratio of the Fermi energy and characteristic electrostatic attraction between graphene carriers and its gate images. Taking into account magnetic inductance we found $v_S^2 = v_0^2\left(\kappa+1\right)\Big/2\left(\kappa+\left(\varepsilon_{ox}/2\right)\left(v_0^2/c^2\right)\right)$ that implies the light velocity in media as propagation speed $v_S \cong c/\sqrt{\varepsilon_{ox}}$ for very thick isolator layers ($\kappa \to 0$). The wave equation is invariant under "pseudo-Lorentz" transformations (where the signal velocity plays a role of the light velocity), which describe plasmon retardation, provide causality and conserve "pseudo-interval" between events $s^2 = r^2 - v_S^2 t^2$, wavelet form, etc. The fundamental solution of Green's function for 2D wave equation has "relativistic-invariant" form

$$u\left(r,t\right) = \frac{1}{2\pi v_S}\frac{\theta\left(v_S t - r\right)}{\sqrt{v_S^2 t^2 - r^2}} \tag{10}$$

In contrast to 3D electrodynamics (where perturbation propagates as inflated sphere $\propto \delta\left(c^2 t^2 - r^2\right)$ according to the Huygens' principle) the fundamental solution of 2D (as well as 1D) wave equation for plasmons has no trailing edge of the signal. Interestingly that the substitution $u\left(r,t\right) = w\left(r,t\right)\exp\left(-t/2\tau\right)$ leads to a new equation [4]

$$\left(v_S^2\nabla^2 - \partial_t^2\right)w\left(r,t\right) = -\left(4\tau^2\right)^{-1} w\left(r,t\right)\,, \tag{11}$$

which is nothing but the classic (2+1)D Klein-Gordon equation with the "light" velocity v_S and imaginary mass $m = i/2\tau$ which displays the slow decay of long-wavelength fluctuations (plasmons). Eq.7 allows also an exact explicit fundamental solution for one-dimensional geometry relevant for long transmission lines

$$u\left(x,t\right) = \frac{1}{2v_S}\theta\left(t/v_S - |x|\right)e^{-\frac{t}{2\tau}} I_0\left(\frac{\sqrt{t^2 - x^2/v_S^2}}{2\tau}\right) \tag{12}$$

where $\theta(z)$ is the Heaviside unit step function and $I_0(z)$ is the modified Bessel function. This general solution exhibits explicitly a crossover between dissipative diffusion-like and lossless wave-like types of signal propagation (see Fig.1).

References

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[3] G.I. Zebrev et al. "Influence of interface traps and electron-hole puddles on quantum capacitance and conductivity in graphene field-effect transistors" arXiv: cond-mat 1011.5127

[4] A. Polyanin, "Handbook of Linear PD Equations for Scientist and Engineers", Chapman&Hall CRC, 2002

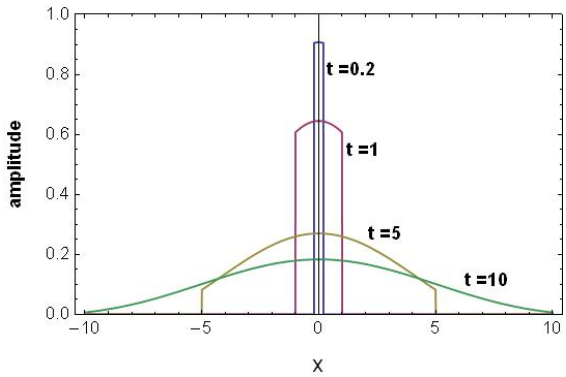

Fig.1a Crossover between wave-like $t/\tau \ll 1$ and diffusive-like ($t/\tau \gg 1$)relaxation according to eq.12.

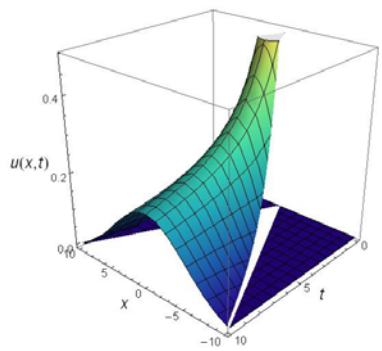

Fig.1b 1D perturbation spreading within the "pseudo-light" cone.