Anomalous Thermal Transport in Low Dimensional Nanostructures

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The study of thermal transport in low dimensional nano scale structures is important for both fundamental research and industrial applications. On the one hand, the low dimensional nanostructures such as graphene, nanowires, and nanotubes provide a test bed for the conjectures and hypothesis proposed in the last two decades for heat transport in two dimensional (2D) and one dimensional (1D) systems [1]. On the other hand, low dimensional nano structured materials have been found to be an ideal candidates to realize phononic functions such as thermal rectifier [2].

In bulk material, heat conduction is governed by Fourier’s law as:

\[ J = -\kappa \nabla T \]  

where \( J \) is the local heat flux and \( \nabla T \) the temperature gradient, \( \kappa \) is the thermal conductivity which is size independent. This is based on the assumption that phonons transport diffusively. However, for the low dimensional systems, in particular 1D systems, except for a simple harmonic oscillator chains, we don’t have any rigorous mathematical proof if the normal diffusion process can happen. Therefore it is still an open question whether the Fourier law is valid in 1D and 2D systems. Especially the sufficient and necessary condition for Fourier’s law is not clear yet [1].

In this talk, I will demonstrate that heat transfers in 1D and 2D systems are significantly from bulk material.

1. Both numerically and experimentally show that thermal conductivity in 1D and/or quasi 1D system such as nanowire and nanotube is not a constant. It depends on the system length as [3-4] (Fig 1 for silicon nanowire and Fig 2 for nanotube):

\[ \kappa \sim L^\beta, \quad 0<\beta<1. \]

2. Thermal conductivity in single layer suspended graphene depends on the length logarithmically when the width is fixed [5].

In the last part, I will present our recent mathematical theory which bridges the anomalous thermal conductivity to anomalous energy diffusion [9,10].

References:
2. N.-B Li, J. Ren, L Wang, G Zhang, P Hanggi, and B Li, Rev. Mod. Phys, in press (2012)
5. X-F Xu et al Nature Materials (submitted)
10. S Liu, J Ren, N.-B Li, and B Li, Phys. Rev. Lett (submitted)
**Figure 1.** The dependence of thermal conductivity of SiNWs on the longitude length $L_z$. The results by Nose-Hoover method coincide with those by Langevin methods, indicating that the results are independent of the heat bath used. The black solid curves are the power law fitting curves (linear in log-log scale). For more details see Ref [3].

**Figure 2.** Left: A SEM image of a thermal conductivity test fixture with a nanotube after five sequences of $(\text{CH}_3)_3\text{(CH}_3\text{C}_5\text{H}_4)\text{Pt}$ deposition. The numbers denote the $n$th deposition. The inset shows the SEM image after the first $(\text{CH}_3)_3\text{(CH}_3\text{C}_5\text{H}_4)\text{Pt}$ deposition. Right: Normalized thermal resistance vs normalized sample length for CNT (solid black circles), best fit assuming $\beta=0.6$ (open blue stars), and best fit assuming Fourier’s law (open red circles). For more details see Ref [4].

**Figure 3: **Length dependence of $\kappa$ at room temperature (black solid circles) of single suspended graphene. The black linear line is a guide to the eyes. The black arrow and dashed line indicate the phonon mean free path. The red solid honeycombs and squares are plotted by extracting the data calculated by Lindsay *et al.*[6] and Nika *et al.*[7], respectively. Note that this $\sim \log L$ is only valid when $L \gg \lambda$ and deviations from $\sim \log L$ in the shortest samples is expected [8]. Insert: illustration of $\log \kappa \sim \log L$ scaling behavior for 1D, 2D and 3D systems, where thermal conductivity scales as $\sim L^{0.3}$, $\sim \log L$ and constant, respectively.