## Zero-energy states in graphene waveguides, quantum dots and rings

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There is a widespread belief that electrostatic confinement of graphene charge carriers, which resemble massless Dirac fermions, is impossible as a result of the Klein paradox. We show that full confinement is indeed possible for zero-energy states in pristine graphene. We present exact analytical solutions for the zero-energy modes of two-dimensional massless Dirac fermions confined within a smooth one-dimensional potential  $V(x) = -\alpha/\cosh(\beta x)$  [1]. This potential provides a reasonable fit for the potential profiles of existing top-gated graphene structures [2-5]. A simple relationship between the characteristic strength and the number of modes within the potential is found. An experimental setup is proposed for the observation of these modes (see Fig.1). Thus, we present a solution to obtaining on/off behavior within graphene, a major obstacle to device realization.

A new numerical method, [6] based on the variable-phase method [7] has been developed to find the number of fully confined zero-energy modes in any smooth potential, decaying at large distances faster than the Coulomb potential. The method allows one to reformulate the Dirac-Weyl equation governing the charge carriers in graphene into a nonlinear, first-order differential for reflection coefficient. The method is numerically efficient and can easily be used to evaluate the conductivity of a channel formed by a realistic top-gate potential.

We also show that full confinement is possible for zero-energy states in electrostatically-defined quantum dots and rings with smooth potential profiles. Again, analytic solutions are found for a class of model potentials [8]. These exact solutions allow us to draw conclusions on general requirements for the potential to support fully confined states, including a critical value of the potential strength and spatial extent. We have also developed our numerical method so that we can handle two-dimensional potentials and so treat scattering problems. The implications of fully-confined zero-energy states for STM measurements and minimal conductivity in graphene are discussed.

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## Figures



Fig. 1: (a) A schematic diagram of a Gedanken experiment for the observation of localized modes in graphene waveguides, created by the top gate ( $V_{TG}$ ). The Fermi level is set using the back gate ( $V_{BG}$ ) to be at  $\varepsilon_F = 0$ . (b) The electrostatic potential created by the applied top gate voltage. The plane shows the Fermi level position at  $\varepsilon_F = 0$ .