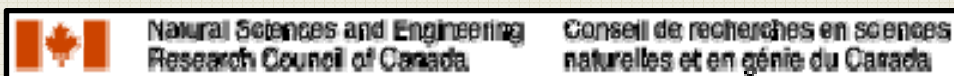


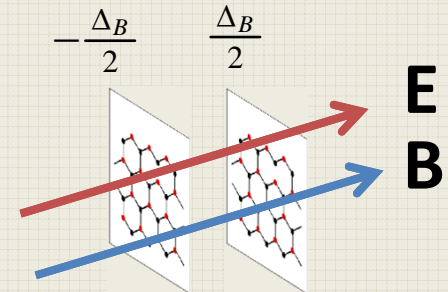
# *Optical properties of Wigner crystal and helical state in bilayer graphene*

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Département de physique  
Université de Sherbrooke



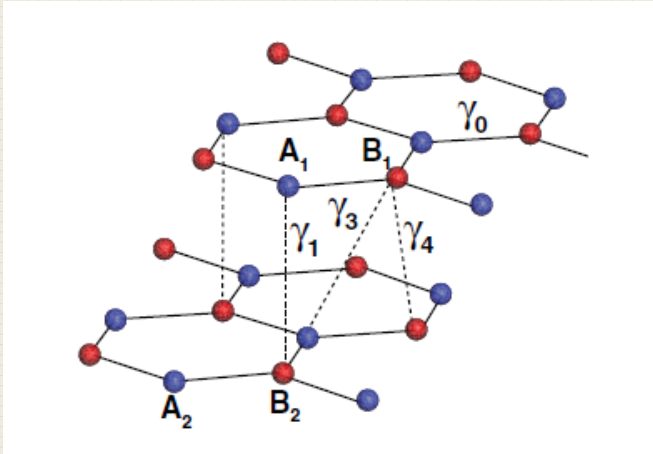
# Outline

Non-collinear orbital pseudomagnetic states can exist in Landau level  $N=0$  of a **biased** graphene bilayer in a magnetic field. They can be detected by optical experiments.



- Origin of the orbital pseudospin in biased bilayer graphene.
- Phase diagram for non-collinear pseudomagnetic states.
- Optical properties: absorption and Kerr effect.

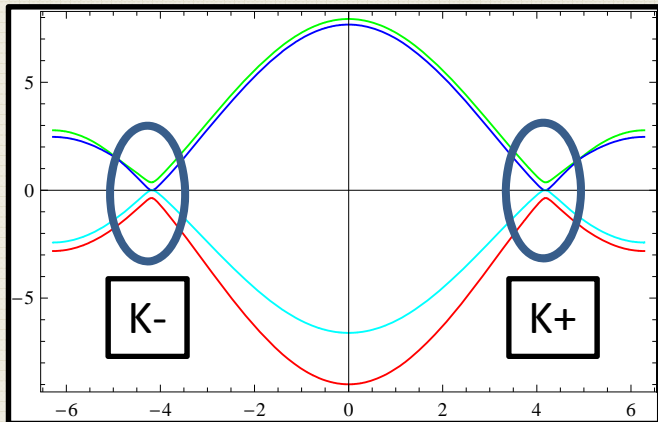
# Bernal-stacked graphene bilayer



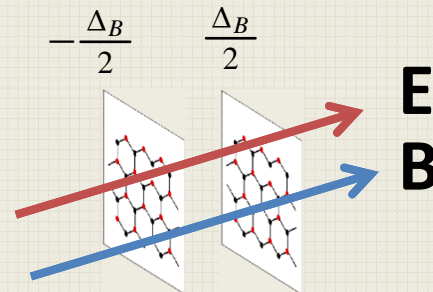
A1-B2: high-energy sites  
A2-B1: low-energy sites

$\gamma_0 = 3.12$  eV n.n intralayer hopping

$\gamma_1 = 0.38$  eV n.n interlayer hopping



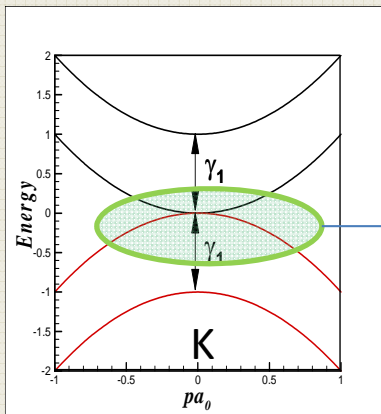
Band structure and the two valleys





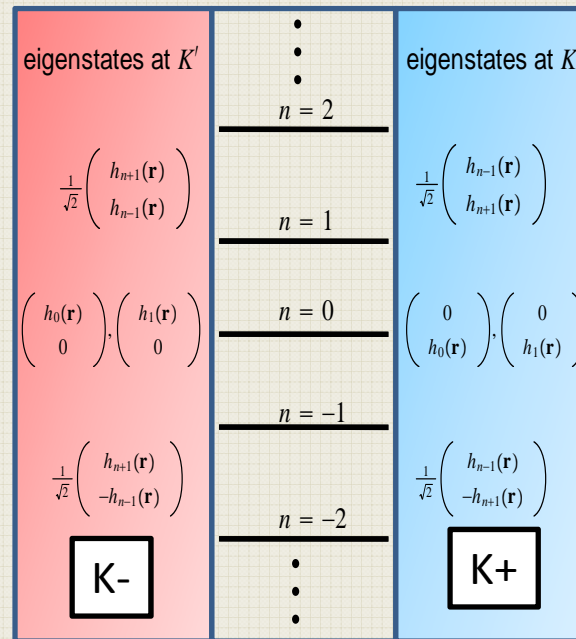
# BLG in a magnetic field: Landau levels

NO BIAS



$$E_n = \pm \hbar \omega_c^* \sqrt{|n|(|n| + 1)}$$

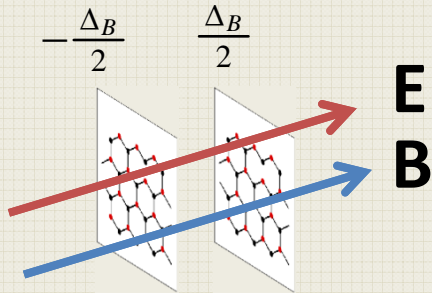
$$n = \dots, -3, 2, -1, 0, 1, 2, 3, \dots$$



$$\begin{pmatrix} A_2 \\ B_1 \end{pmatrix} \begin{matrix} \text{Bottom layer} \\ \text{Top layer} \end{matrix}$$

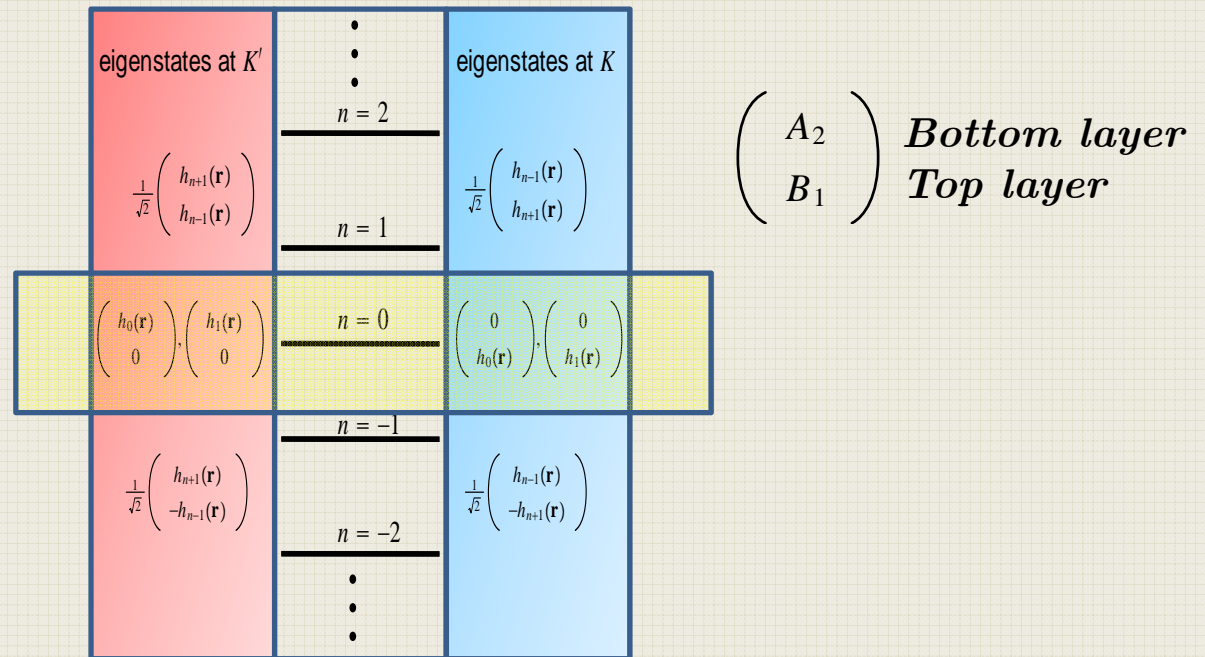
Two-component model  
amplitudes on low-energy sites

# N=0 Landau level degeneracy



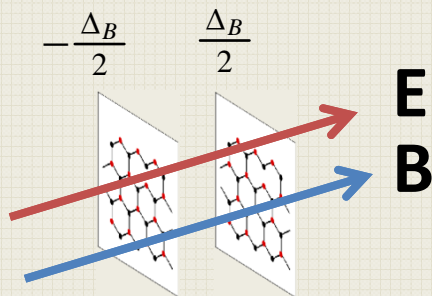
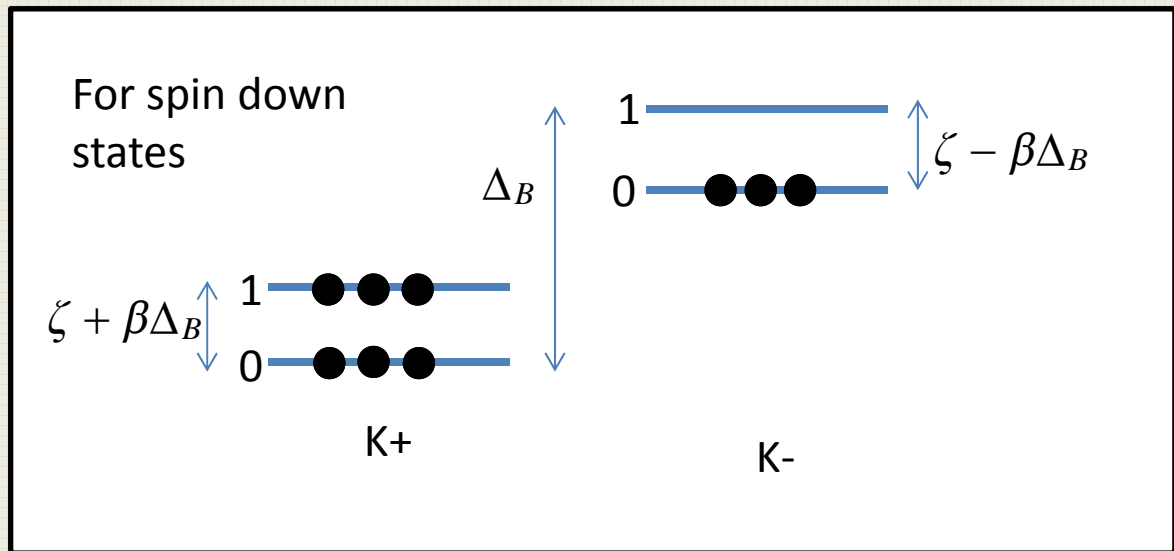
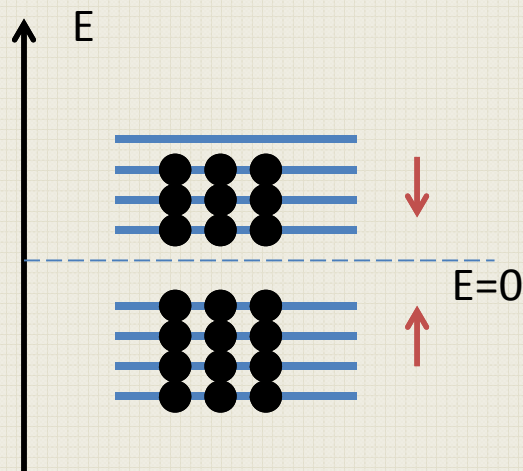
$$h_n(\mathbf{r}) = \frac{1}{\sqrt{L_y}} e^{-iXy/\ell^2} \varphi_n(x - X) \quad \text{orbital}$$

Valley and layer indices are related in N=0



Landau level N=0 has 8 sub-levels: 2(spin) X 2(valleys) X 2(orbitals)

# Special case: $N=0$ and $\nu=3$



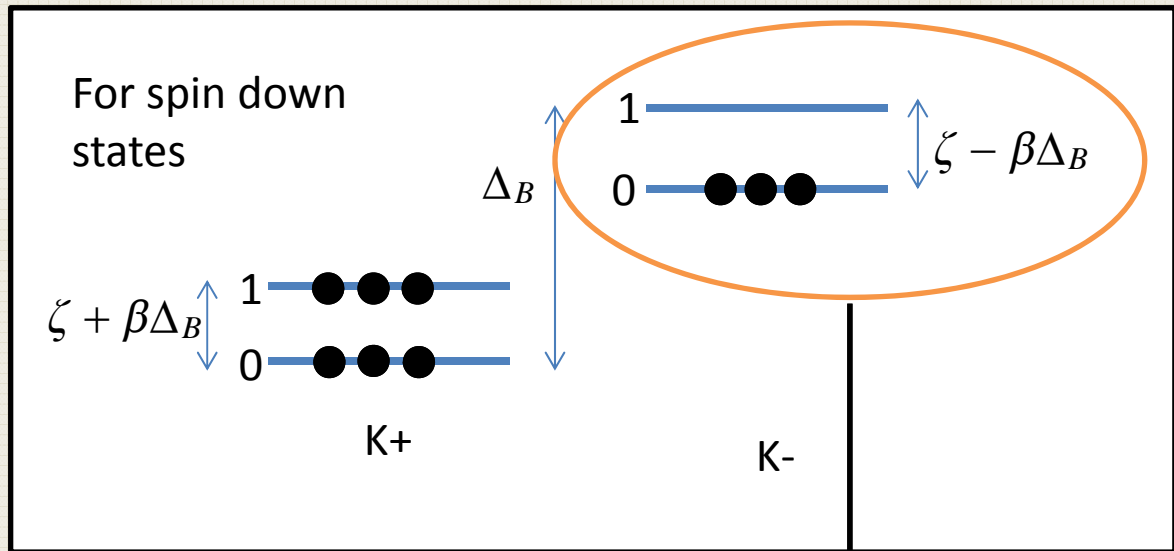
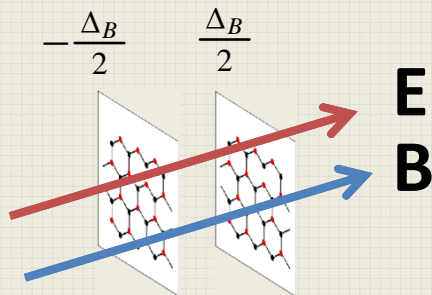
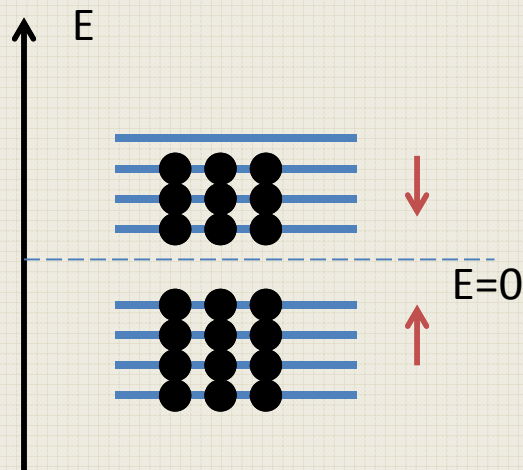
$$\zeta = \beta \left( 2 \frac{\gamma_1 \gamma_4}{\gamma_0} + \delta_0 \right),$$

$$\beta = \frac{\hbar \omega_c^*}{\gamma_1} = 7.24 \times 10^{-3} B[\text{T}],$$

$$\begin{aligned} \zeta &= 0.39 B[\text{T}] \text{ meV}, \\ \Delta_Z &= 0.12 B[\text{T}] \text{ meV}. \end{aligned}$$



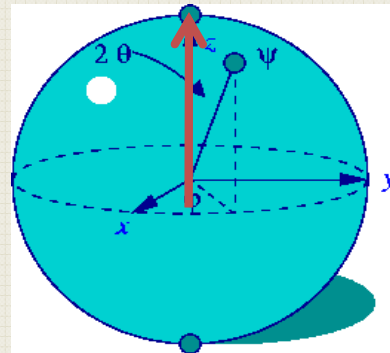
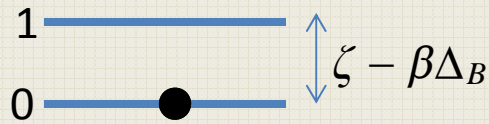
# A two-level system



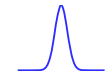
The 0-1 gap can be made negative !

Our two-level system:  
the gap is controlled by the electric field between the layers.

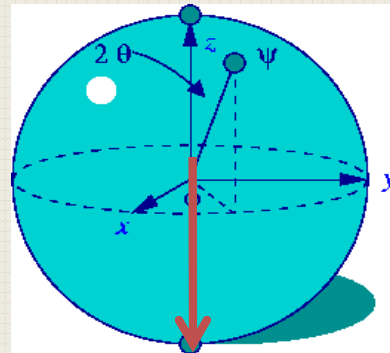
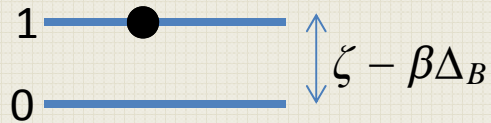
# Orbital pseudospin



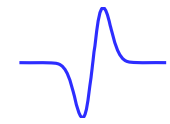
*orbital  $n=0$*



$$\begin{pmatrix} 0 \\ \varphi_0 \end{pmatrix}$$



*orbital  $n=1$*

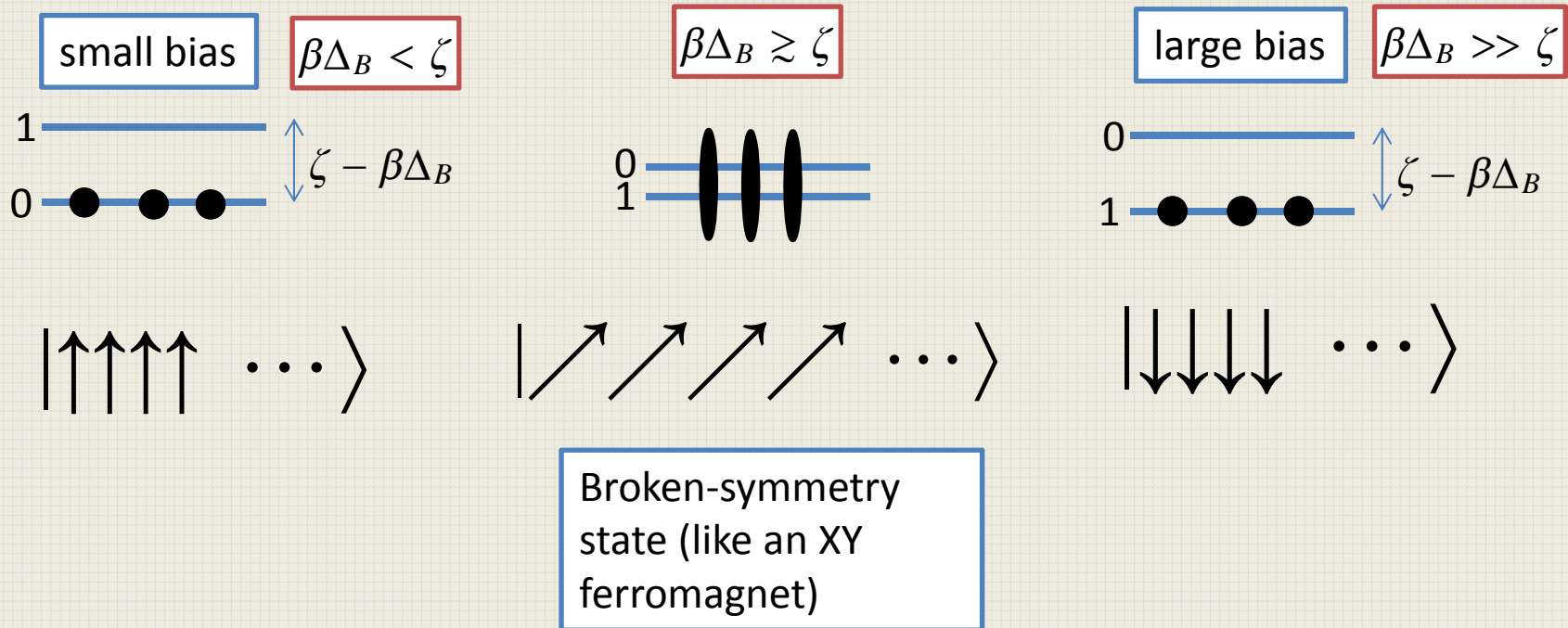


$$\begin{pmatrix} 0 \\ \varphi_1 \end{pmatrix}$$

Quantum mechanics allows any linear combinations of these two states so that the orbital pseudospin can point anywhere on the Bloch sphere.



# Quantum Hall ferromagnetism with orbital pseudospin



Coulomb exchange interaction is more negative for electrons in  $n=0$  than for electrons in  $n=1$

# General Hartree-Fock Hamiltonian

$$E_{HF} = -\frac{11}{32} \sqrt{\frac{\pi}{2}} - \frac{1}{2} \beta \Delta_B + \frac{1}{\sqrt{2} \ell e} \beta \left( \Delta_B - \frac{1}{2} \Delta_B^{(1)} \right) p_z(0) \text{ « Zeeman » coupling}$$

$$+ \frac{1}{4e^2 \ell^2} \sum_{\mathbf{q}} \mathbf{p}_{\parallel}(-\mathbf{q}) \cdot [a(q)\mathbf{I} + b(q)\Lambda(\mathbf{q})] \cdot \mathbf{p}_{\parallel}(\mathbf{q})$$

$$+ \frac{1}{4e^2 \ell^2} \sum_{\mathbf{q}} c(q) p_z(-\mathbf{q}) p_z(\mathbf{q})$$

**a and c: nonlocal exchange interactions**  
**b: dipole-dipole electrostatic interaction**

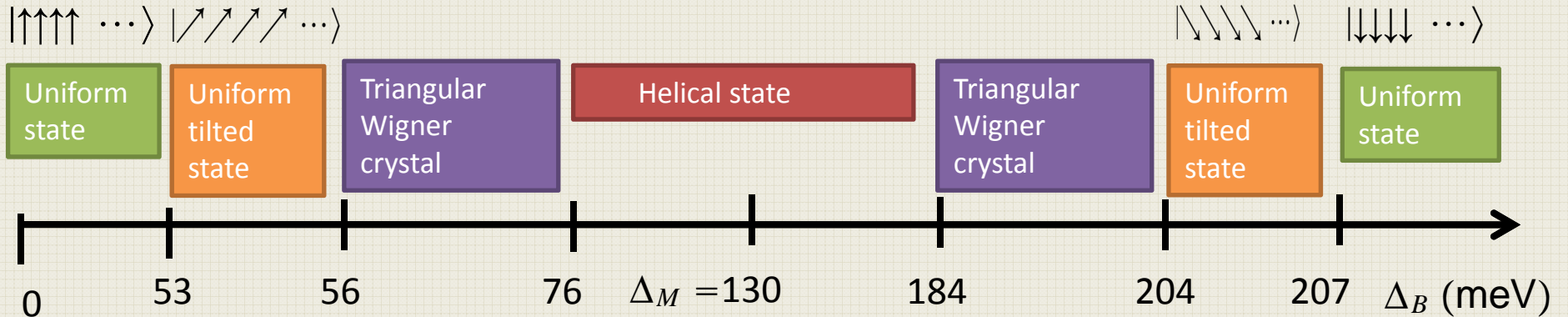
$$+ \frac{i}{8e^2 \ell^2} \sum_{\mathbf{q}} d(q) (\hat{\mathbf{z}} \times \hat{\mathbf{q}}) \cdot (\mathbf{p}(-\mathbf{q}) \times \mathbf{p}(\mathbf{q})).$$

$\sim D \int d\mathbf{r} (\mathbf{p} \cdot (\nabla \times \mathbf{p}))$

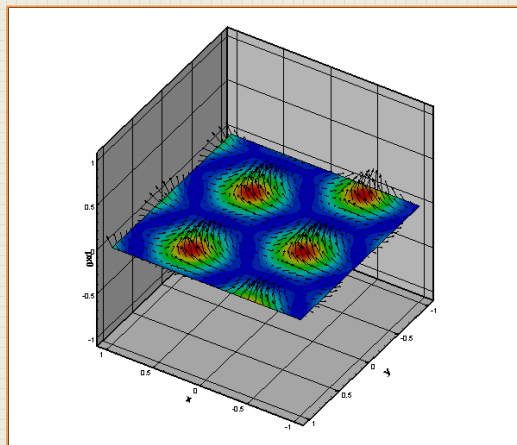
**p= orbital pseudospin**

**Dzyaloshinski–Moriya interaction**  
**(from Coulomb interaction, not spin-orbit)**

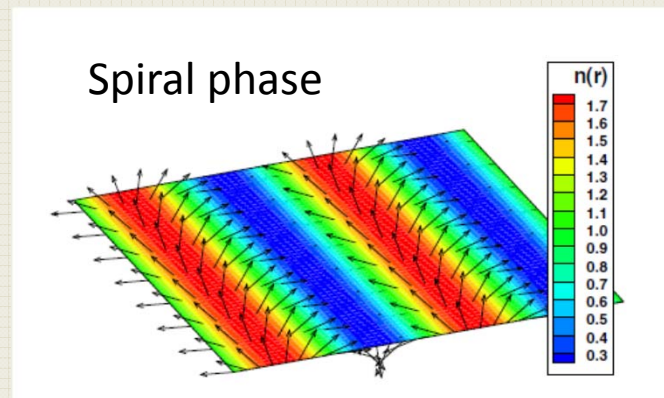
# Phase diagram for $\nu=3$



$B = 10 \text{ T}$   
 $\epsilon = 5$



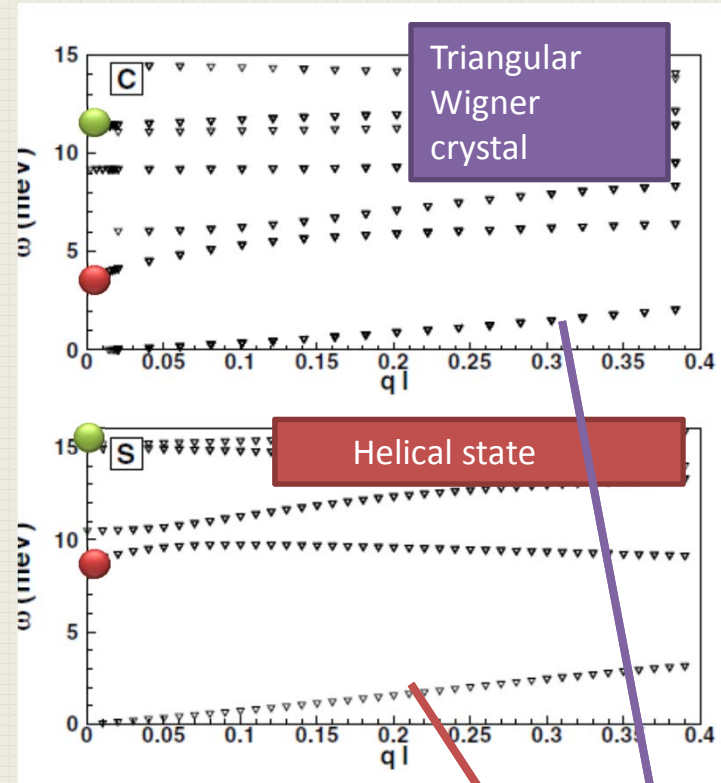
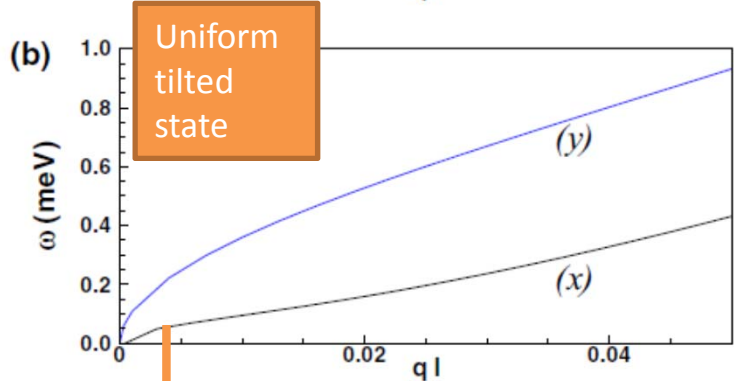
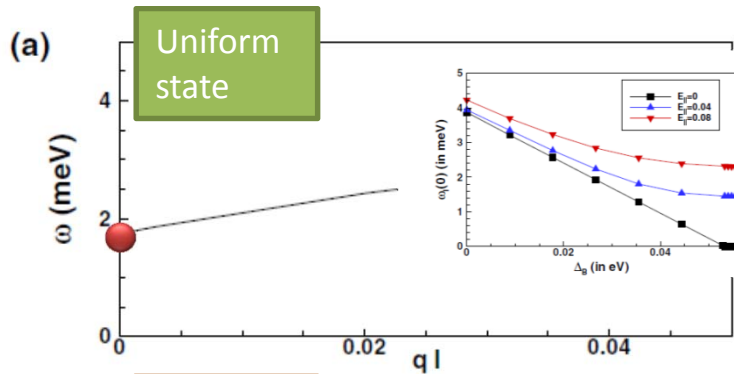
Wigner crystal  
with meron-like  
pseudospin texture



The electronic density  
is also modulated  
in space.



# Magnetoexciton modes

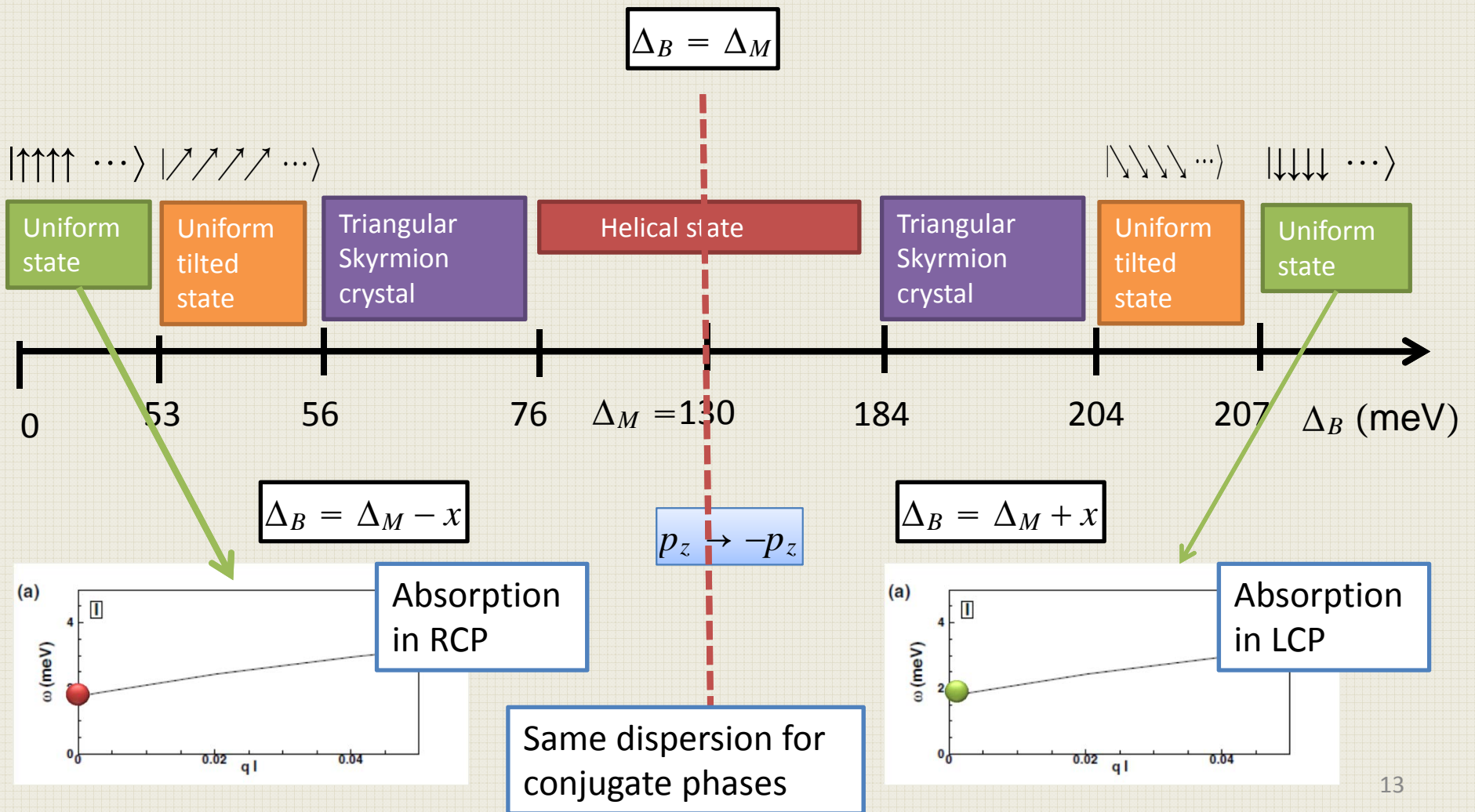


Goldstone mode with anisotropic dispersion

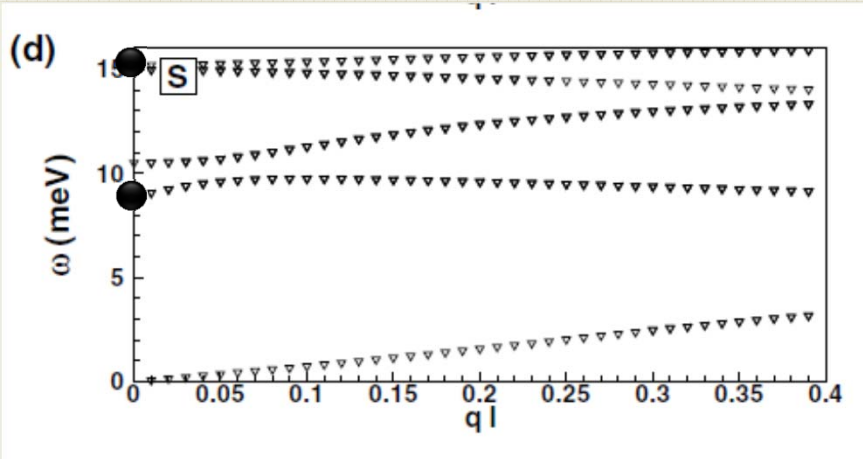
$$3.85 \text{ meV} \Rightarrow 9.3 \times 10^{11} \text{ Hz}$$

Gapless phonon mode in the WC and helical phases

# Symmetry of the phase diagram: conjugate states



# Optical absorption in helical state



Modes are not fully circularly polarised.  
in helical state

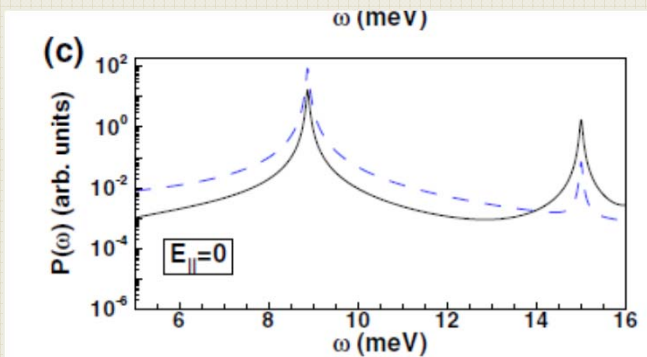
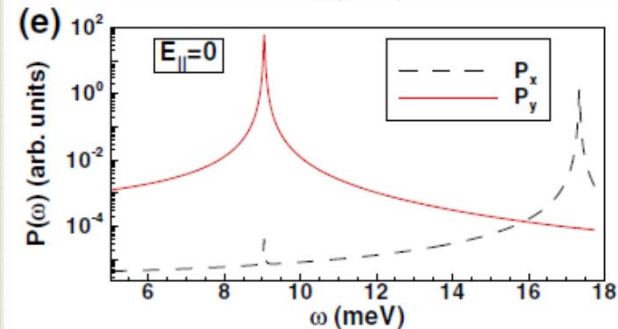
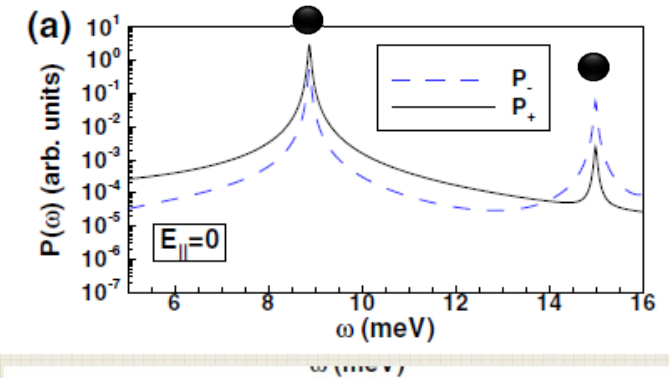
linear polarisation for  $\Delta_B = \Delta_M$

Symmetry RCP-LCP for conjugate  
phases

$$\Delta_B = \Delta_M - \chi$$

$$\Delta_B = \Delta_M$$

$$\Delta_B = \Delta_M + \chi$$





# Kerr effect

Modes that are active in absorption also show a Kerr effect

Uniform  
state

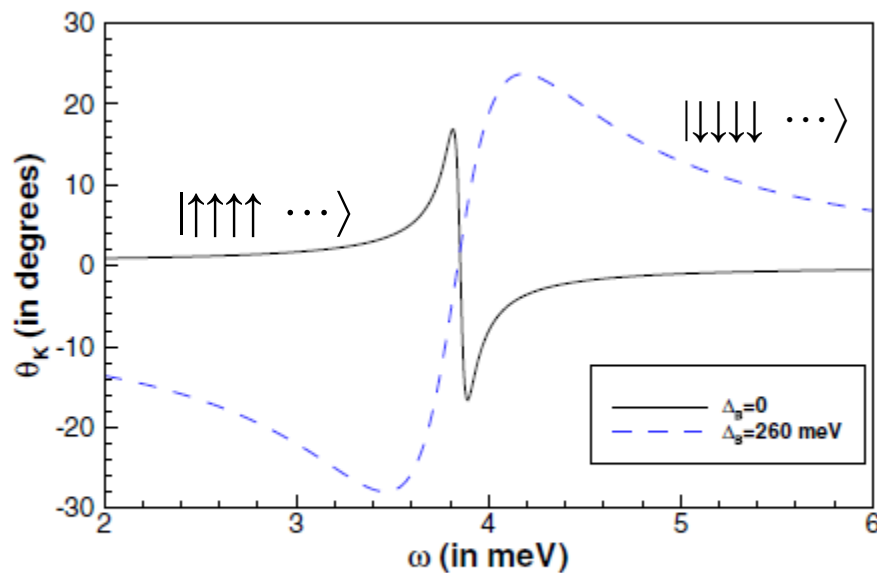


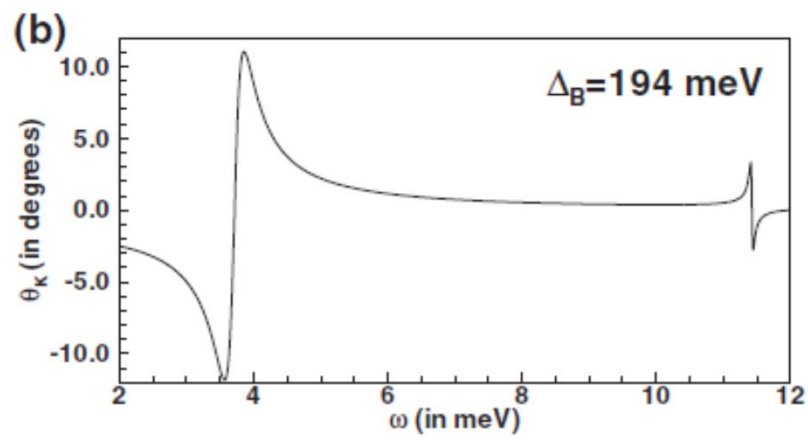
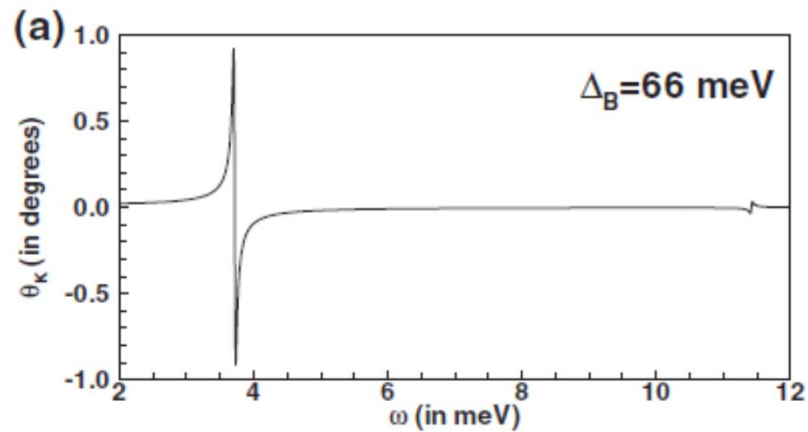
FIG. 12. (Color online) Kerr angle for conjugate biases  $\Delta_B = 0$  and  $\Delta_B = 260$  meV in the  $I$  and  $I^*$  phases.

Polarisation angle rotates in opposite directions for conjugate phases.

Also in the WC and helical phases.

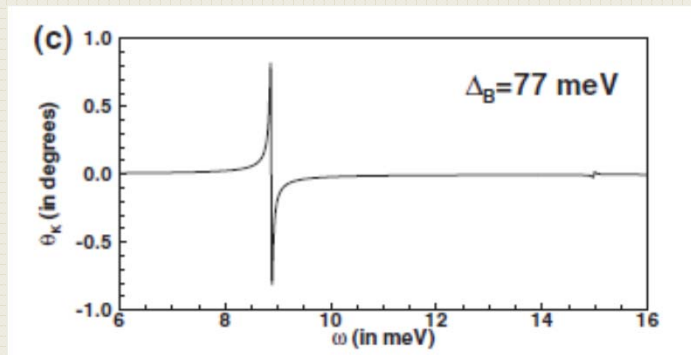
We have not included disorder so that we cannot obtain the numerical value of the Kerr angle.

# Kerr effect: skyrmion crystal

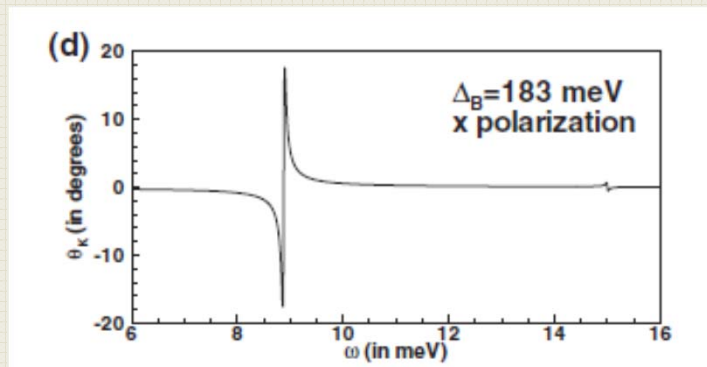


Polarisation angle rotates in the opposite direction in the conjugate phase.

# Kerr effect: helical state



Polarisation angle rotates in opposite directions for conjugate phases.



No Kerr effect at  $\Delta_B = \Delta_M$

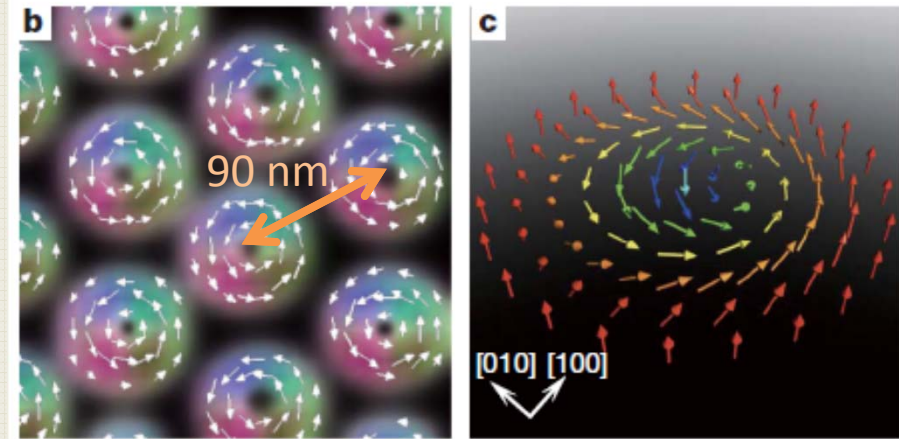
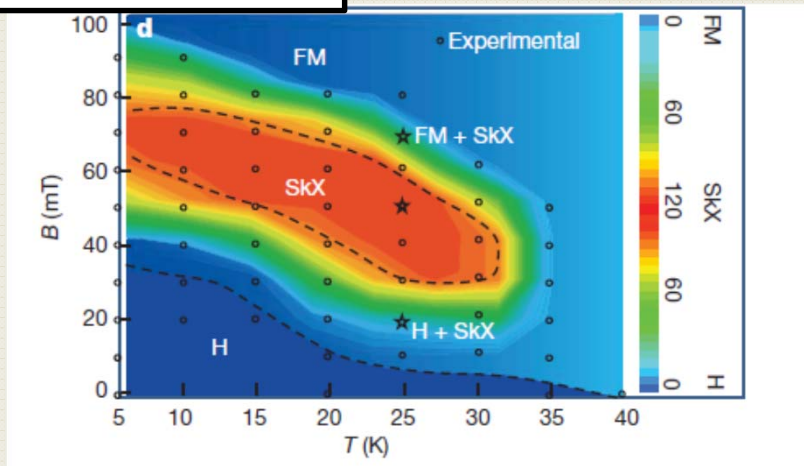


# Conclusion

- ❖ At some special filling factors, the biased Bernal-stacked bilayer graphene behave as an helical ferromagnet.
- ❖ Several nonuniform phases are obtained as the bias is increased.
- ❖ All phases but one (the tilted state) have gapped magnetoexcitonic states, some of which are active in optical absorption and Kerr rotation. Conjugate phases are active in opposite circular polarisations and their Kerr angle rotate in opposite directions.

# Non-collinear spin configurations in helical ferromagnets

Magnetic field perpendicular to thin film plane



**Figure 3 | Phase diagrams of magnetic structure and spin textures in a thin film of  $\text{Fe}_{0.5}\text{Co}_{0.5}\text{Si}$ .** a–c, Spin textures observed using Lorentz TEM

Real-space imaging using Lorentz transmission electron microscopy

$$H = \int d\mathbf{r} \left[ \frac{J}{2} (\nabla \mathbf{M})^2 + \alpha \mathbf{M} \cdot (\nabla \times \mathbf{M}) \right]$$

Ferromagnetic exchange  
+ Dzyaloshinskii  
-Moriya interaction

From X. Z. Yu et al., *Nature Letters*, 465, 901 (2010).

We find a similar behavior with bias in Bernal-stacked bilayer graphene : an effective DM interaction arises from the Coulomb interaction and the magnetic moments are replaced by electric dipoles. The different phases are detectable in optical (absorption, Kerr) experiments.